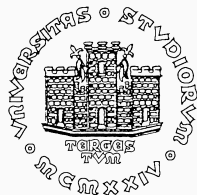


Systems Dynamics

Course ID: 267MI – Fall 2018

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267MI –Fall 2018

Lecture 4

Model identification from data

**Modelling
Identification
Prediction
& filtering**



Disciplines providing tools to **estimate** variables and unknown parameters and to **design models** of natural and artificial systems using **experimental data**.

Why do we need models?

*Constructing models for a slice of reality and studying their properties is really what science is about. The **models** – “the hypotheses”, “the laws of nature”, “the paradigms” – can be of a more or less formal character, but they all have the **fundamental property** that they try **to link observations to some pattern**.*

L. Ljung, T. Glad, “Modeling of Dynamic Systems”, Prentice Hall, 1994

"Transparent box" modeling approach

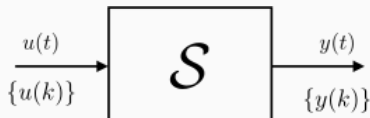
So far, approach undertaken to devise dynamical models:

Inputs ("causes")

$$u(t) \in \mathbb{R}^m$$
$$\{u(k) \in \mathbb{R}^m\}$$

Outputs ("effects")

$$y(t) \in \mathbb{R}^p$$
$$\{y(k) \in \mathbb{R}^p\}$$



Definition of the
"system" entity to
be analysed



Physical laws, a
priori knowledge,
heuristic
considerations,
statistical
evidence, etc.



*Mathematical
models: algebraic
and/or
differential
and/or difference
equations*

A different approach (data-based)



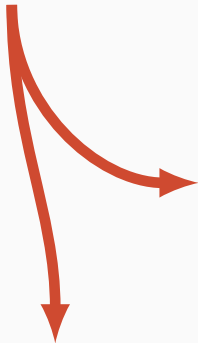
Experimental data, sensor measurements, historical data, etc.

“Black-box” modeling approach

Input, output and disturbance variables are characterized in terms of numerical sequences. These are the data to be used to determine the dynamical model.

“Black-box” modeling approach (identification)

Given: $\{y(k)\}, \{u(k)\}$



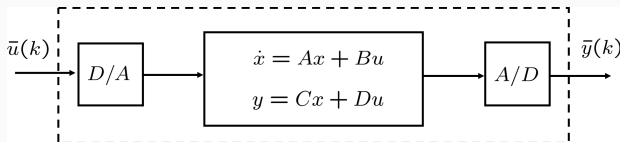
Discrete-time model:

- continuous-time system and data obtained by sampling
- discrete-time system and data inherently discrete-time

Finite-difference equations

Continuous-time system and data obtained by sampling

Linear, time-invariant case:



$$u(t) = \bar{u}(k) \\ t_k \leq t < t_{k+1}$$

Recall the **step-invariant transform**

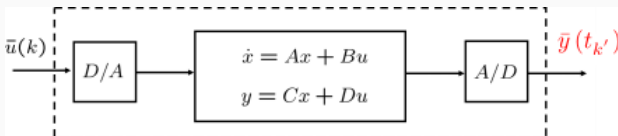
$$\bar{y}(k) = y(t_k)$$

$$\begin{cases} \bar{x}[(k+1)] = \bar{A}\bar{x}(k) + \bar{B}\bar{u}(k) \\ \bar{y}(k) = \bar{C}\bar{x}(k) + \bar{D}\bar{u}(k) \end{cases}$$

Letting $t_{k+1} - t_k = \Delta$

$$\bar{A} = e^{A\Delta} \quad \bar{B} = \int_0^{\Delta} e^{Ar} B dr \quad \bar{C} = C \quad \bar{D} = D$$

Continuous-time system and data obtained by sampling (cont.)



$$u(t) = \bar{u}(k)$$

$$t_k \leq t < t_{k+1}$$

What if the output is sampled at

$t_{k'} \neq t_k$ with $t_k \leq t_{k'} < t_{k+1}$?

$$\bar{y}(k) = y(t_{k'})$$

- Let's recall the expression

$$y(t) = C e^{A(t-t_0)} x(t_0) + C \int_{t_0}^t e^{A(t-\tau)} B u(\tau) d\tau + D u(t)$$

for the movement of the output of a continuous-time LTI system (from “*Fundamentals of Automatic Control*”).

Continuous-time system and data obtained by sampling (cont.)

- Let's consider t_k as initial time instant (i.e. $t_0 = t_k$), the instant $t_{k'}$ as final time instant and let's assume $t_{k'} - t_k = \alpha$. Recall also the stair-wise behavior of the input signal:

$$u(t) = \bar{u}(k), \quad t_k \leq t < t_{k+1}.$$

$$y(t_{k'}) = C e^{A\alpha} x(t_k) + C \left(\int_{t_k}^{t_{k'}} e^{A(t_{k'} - \tau)} B d\tau \right) u(t_k) + Du(t_k)$$

- Substitute $r = t_{k'} - \tau$ into the integral term and rewrite the expression

$$y(t_{k'}) = C e^{A\alpha} x(t_k) + C \left(\int_0^\alpha e^{Ar} dr \right) Bu(t_k) + Du(t_k)$$

- Let's compare with the discrete-time output expression

$$\bar{y}(k) = \bar{C}\bar{x}(k) + \bar{D}\bar{u}(k)$$

Continuous-time system and data obtained by sampling (cont.)

- If $t_{k'} \neq t_k$ then

$$\bar{C} = C e^{A\alpha} \quad \bar{D} = C \left(\int_0^\alpha e^{Ar} dr \right) B + D \quad t_{k'} - t_k = \alpha (< \Delta)$$

- When $t_{k'} = t_k$ obviously

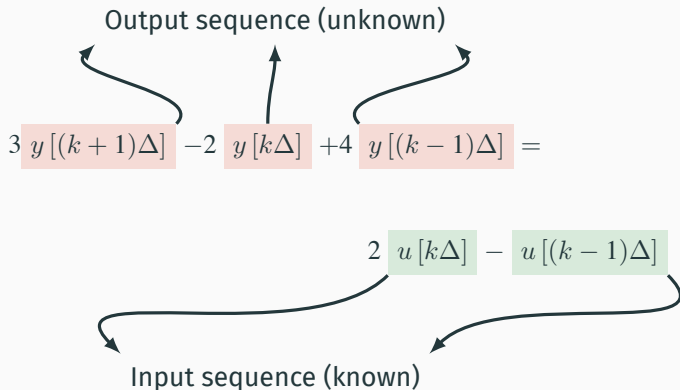
$$\bar{C} = C \quad \bar{D} = D$$

- In both the cases the following expressions hold (remember the *step-invariant transform*)

$$\bar{A} = e^{A\Delta} \quad \bar{B} = \int_0^\Delta e^{Ar} B dr \quad \Delta = t_{k+1} - t_k \quad \forall k$$

“Black box” modeling: an example

As usual, let's assume Δ as the sampling time.



- With sampling-time Δ enhanced:

$$3y[(k+1)\Delta] - 2y[k\Delta] + 4y[(k-1)\Delta] = \\ 2u[k\Delta] - u[(k-1)\Delta]$$

- Compact:

$$3y_{k+1} - 2y_k + 4y_{k-1} = 2u_k - u_{k-1}$$

- Typical framework: **linear finite-difference equations** with constant coefficients.
- General expression takes on the form:

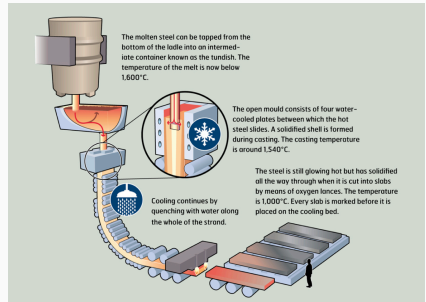
$$a_n y_{k+n} + a_{n-1} y_{k+n-1} + \cdots + a_0 y_k = \\ b_m u_{k+m} + b_{m-1} u_{k+m-1} + \cdots + b_0 u_k$$

with given initial conditions

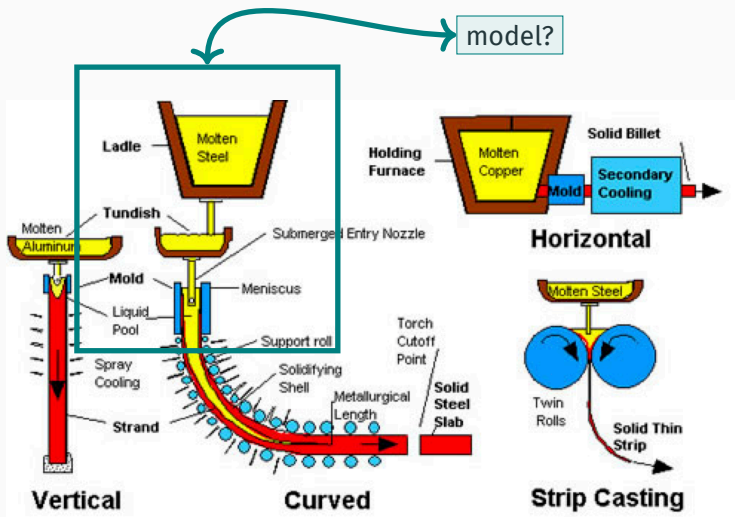
$$\{y_k : k = -n, -(n-1), \dots, 0\}$$

and known input sequence u_k .

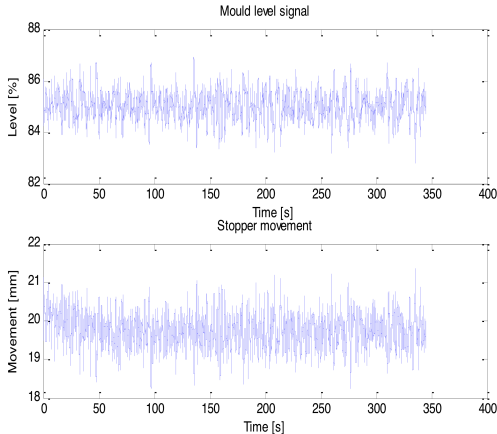
A real application: steel continuous casting



A real application: steel continuous casting (cont.)



A real application: steel continuous casting (cont.)



**“Black-box”
modeling approach**



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END