Systems Dynamics

Course ID: 267MI – Fall 2018

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Lecture 1
Generalities: systems and models

Systems Dynamics

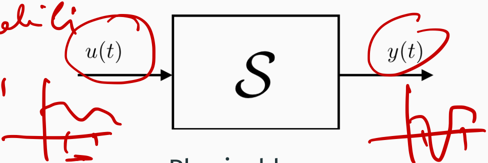
Systems

Inputs ("causes") | Outputs ("effects")

$$u(t) = \begin{bmatrix} u_1(t) \\ \vdots \\ u_m(t) \end{bmatrix} \in \Re^m \qquad y(t) = \begin{bmatrix} y_1(t) \\ \vdots \\ y_m(t) \end{bmatrix} \in \Re^p$$

- manipolabili
- non "

ci sono se dispongo di sensori



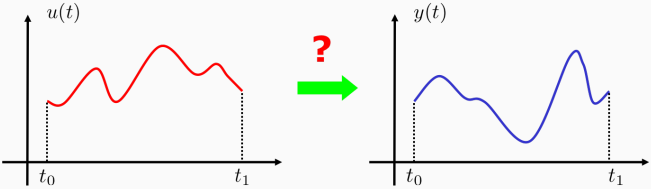
Definition of the "system" entity to be analysed $\Longrightarrow$ Physical laws, a priori knowledge, heuristic considerations, statistical evidence, etc. $\Longrightarrow$ *Mathematical models*: algebraic and/or differential and/or difference equations

Dynamic Systems Described by State Equations

Dynamic Systems

Recalling from the *Fundamentals in Control* course

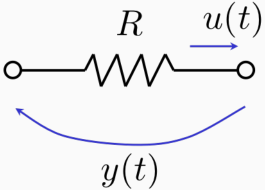
What is the meaning of ”Dynamic”?



Can $y(t)$ be determined in a unique way?

If the answer is ”NO” $\Longrightarrow$ The system is a **dynamic system**

Dynamic Systems: Examples

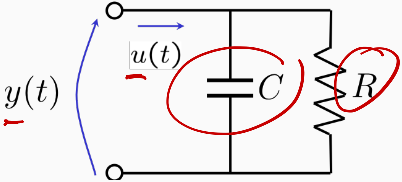


Deciso: (corrente)

- u(t) ingresso
- y(t) uscita (tensione)

$$y(t) = R \cdot u(t)$$

The system is **NOT** dynamic

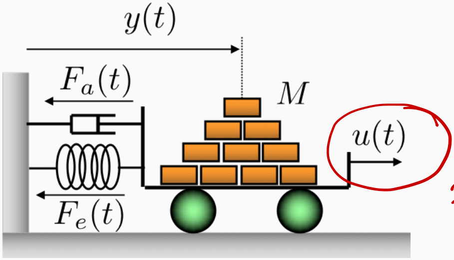


- $u(t),\ t \in [t_0, t_1]$
- $y(t_0)$

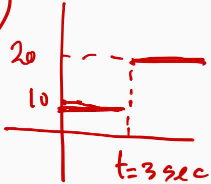
$$\Longrightarrow y(t),\ t \in [t_0, t_1]$$

The system is **dynamic**

Dynamic Systems: Examples



$y(3)$
$\dot{y}(3)$



$$\left.\begin{array}{l} u(t),\ t \in [t_0, t_1] \\ y(t_0) \\ \dot{y}(t_0) \end{array}\right\} \implies y(t),\ t \in [t_0, t_1]$$

t_0

The system is **dynamic**

State variables: a qualitative definition

State variables

Variables to be known at time $t = t_0$ in order to be able to determine the output $y(t)$, $t \geq t_0$ from the knowledge of the input $u(t)$, $t \geq t_0$:

$$x_i(t),\ i = 1, 2, \ldots, n \qquad \text{(state variables)}$$

$\cdots$ In more **rigorous** terms $\Longrightarrow$

Dynamic Systems: Formal Definitions

A **dynamic system** is an abstract entity defined in axiomatic way:

$$\mathcal{S} = \{T, U, \Omega, X, Y, \Gamma, \varphi, \eta\}$$

- T : set of **time instants** provided with an order relation
- U : set of admissible **input** values
- Ω : set of admissible **control functions**
- X : set of admissible **state** values
- Y : set of admissible **output** values
- Γ : set of admissible **output functions**

Dynamic Systems: Formal Definitions (cont.)

State transition function:

istante attuale

" iniziale

$$\varphi : T \times T \times X \times \Omega \mapsto X \quad \Longrightarrow \quad x(t) = \varphi(t, t_0, x_0, u(\cdot))$$

stato iniz.

funzione d'ingresso applicata

1. **Consistency:** $\varphi(t_0, t_0, x_0, u(\cdot)) = x_0\,,\ \forall\,(t_0, x_0, u(\cdot)) \in T \times X \times \Omega$
2. **Irreversibility:** φ is defined $\forall\, t \geq t_0,\ t \in T$
3. **Composition:**

$$\varphi(t_2, t_0, x_0, u(\cdot)) = \varphi(t_2, t_1, \overbrace{\varphi(t_1, t_0, x_0, u(\cdot))}^{x(t_1)}, u(\cdot))$$
$$\forall\,(t_0, u(\cdot)) \in T \times \Omega\,,\ \forall\, t_0, t_1, t_2 \in T : t_0 < t_1 < t_2$$

4. **Causality:**

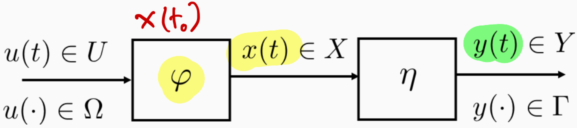
$$u'_{[t_0,t)}(\cdot) = u''_{[t_0,t)}(\cdot) \Longrightarrow \varphi(t, t_0, x_0, u'(\cdot)) = \varphi(t, t_0, x_0, u''(\cdot)),$$
$$\forall (t, t_0, x_0) \in T \times T \times X$$

Dynamic Systems: Formal Definitions (cont.)

Output function:

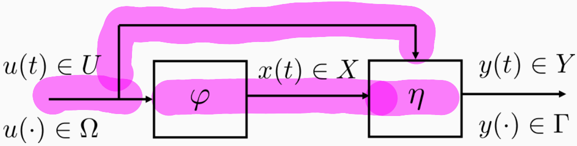
- Case 1: **strictly proper** system:

$$\eta:\ T \times X \mapsto Y \implies y(t) = \eta(t, x(t)),\ \forall t \in T$$



- Case 2: **non strictly proper** system:

$$\eta:\ T \times X \times U \mapsto Y \implies y(t) = \eta(t, x(t), u(t)),\ \forall t \in T$$



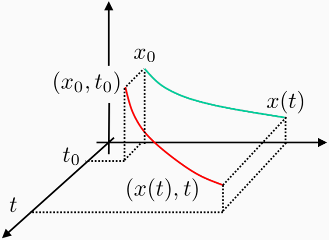
Dynamic Systems: Formal Definitions (cont.)

$(x, t) \in X \times T$ is defined as **event**

Given:

- (x_0, t_0) initial event
- $u(\cdot)$ input function

One has:



$\varphi(\cdot, t_0, x_0, u(\cdot))$ state movement

$\varphi(t, t_0, x_0, u(\cdot)),\ t \geq t_0$ state trajectory

$\eta(\cdot, \varphi(\cdot, t_0, x_0, u(\cdot)))$ output movement

$\eta(\cdot, \varphi(\cdot, t_0, x_0, u(\cdot)))$ output trajectory

Dynamic Systems: Formal Definitions (cont.)

$\bar{x} \in X$ is an **equilibrium state** if $\forall\, t_0 \in T,\ \exists\, u(\cdot) \in \Omega$ such that

$$\varphi(t, t_0, \bar{x}, u(\cdot)) = \bar{x}\,,\ \forall\, t \geq t_0,\ t \in T$$

$\bar{y} \in Y$ is an **equilibrium output** if $\forall\, t_0 \in T,\ \exists \bar{x} \in X,\ \exists\, u(\cdot) \in \Omega$ such that

$$\eta(t, \varphi(t, t_0, \bar{x}, u(\cdot))) = \bar{y}\,,\ \forall\, t \geq t_0,\ t \in T$$

Notice that, in general:

- the specific input function $u(\cdot) \in \Omega$ depends on the choice of the initial time-instant $t_0 \in T$
- the fact that the state of a dynamic system is at equilibrium does not imply that the output is at equilibrium as well, unless $\eta(t, x(t))$ does not depend explicitly on time (in which case, the output function takes on the form $\eta(x(t))$)

Dynamic Systems: Formal Definitions (cont.)

- A dynamic system is **invariant** if T is an additive algebraic group and $\forall\, u(\cdot) \in \Omega$, $\forall\, \tau \in T$, letting $u^\tau(t) := u(t-\tau) \in \Omega$, it follows that

$$\begin{cases} \varphi(t, t_0, x_0, u(\cdot)) = \varphi(t+\tau, t_0+\tau, x_0, u^\tau(\cdot))\,, \ \forall\, t, \tau \in T \\ y(t) = \eta(t, x(t)) \end{cases}$$

- A dynamic system is **discrete-time** if T is isomorphous with $\mathbb{Z}$
- A dynamic system is **continuous-time** if T is isomorphous with $\mathbb{R}$
- A dynamic system is **finite-dimensional** (**lumped-parameter**) if U, X, Y are finite-dimensional vector spaces
- A dynamic system is **infinite-dimensional** (**distributed-parameter**) if U, X, Y are infinite-dimensional vector spaces

Interconnection of Dynamic Systems

We consider **interconnected systems**

$$\mathcal{S} = \{T, U, \Omega, X, Y, \Gamma, \varphi, \eta\}$$

composed of N subsystems

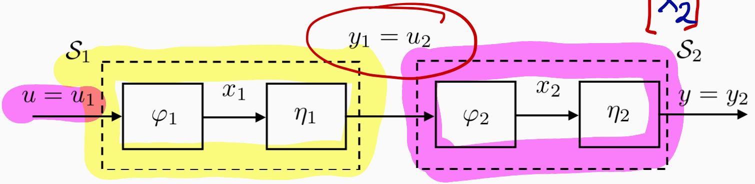
$$\mathcal{S}_i = \{T_i, U_i, \Omega_i, X_i, Y_i, \Gamma_i, \varphi_i, \eta_i\}\ ,\ \ i = 1, 2, \ldots N$$

interacting with each other through their external variables such as inputs $u_i(\cdot) \in \Omega_i$ and outputs $y_i(\cdot) \in \Gamma_i$

Assumption. The interconnected system $\mathcal{S}$ satisfies the formal definition of dynamic system

$X = \begin{bmatrix} x_1 \\ \hdashline x_2 \end{bmatrix}$

Cascade interconnection



$$\mathcal{S} = \{T = T_1 = T_2, U = U_1, \Omega = \Omega_1, X = X_1 \times X_2, Y = Y_2, \Gamma = \Gamma_2\}$$

$$\begin{cases} (x_1(t), x_2(t)) = (\varphi_1(t, t_0, x_1(t_0), u(\cdot)), \\ \qquad \varphi_2(t, t_0, x_2(t_0), \eta_1(t, \varphi_1(t, t_0, x_1(t_0), u(\cdot))))) \\ y(t) = y_2(t) = \eta_2(t, x_2(t)) \end{cases}$$