Es. 1 Processo stazionario $E[v(t)] = 0$

$v(t)$ è v. a.

$$\begin{cases} \gamma(0) = 5 \\ \gamma(\pm 1) = 2 \\ \gamma(\pm\tau) = 0 \quad \forall \tau \geq 2 \end{cases}$$

processo di tipo? decorrelazione? spettro?

→ il processo è <u>sicuramente</u> MA(1)

$$y(t) = e(t) + c_1 e(t-1)$$

$$e(\cdot) \sim WN(0, \sigma^2)$$

$$\gamma(0) = (1^2 + c_1^2)\sigma^2 = 5$$

$$\gamma(\pm 1) = c_1 \sigma^2 = 2$$

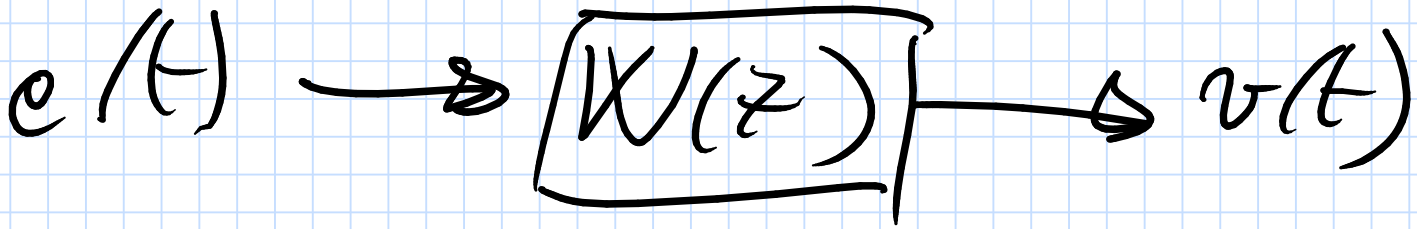
$$\begin{cases} (1 + c_1^2)\sigma^2 = 5 \\ c_1 \sigma^2 = 2 \end{cases}$$

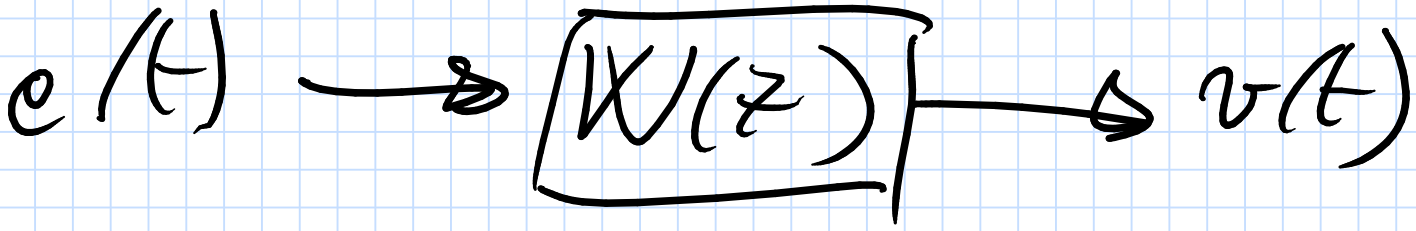
$$\lambda^2 = \frac{2}{c_1} \Rightarrow \left(1+c_1^2\right)\frac{2}{c_1} = 5$$

$$2c_1^2 - 5c_1 + 2 = 0$$

$$c_1 = \begin{cases} +1/2 \Rightarrow \lambda^2 = 4 \\ +2 \Rightarrow \lambda^2 = 1 \end{cases}$$

$$v(t) = e(t) + c_1 e(t-1) \qquad e(\cdot) \sim N(0, \sigma^2)$$



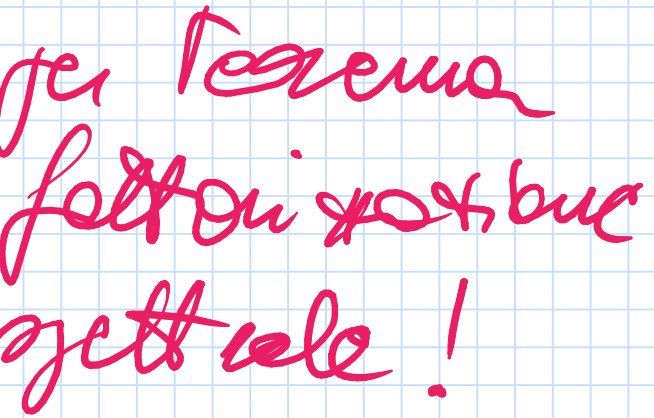


$$W(z) = 1 + c_1 z^{-1} = \frac{z + c_1}{z}$$

? $c_1 = 2$

• $c_1 = 1/2$

← NO per Teorema fattori zero/poli



$$W(z) = \frac{z + 1/2}{z}$$

$c(\cdot) \in WSS(0, 4)$

$$\Gamma_y(\omega) = ? \quad \rightarrow \quad |W(e^{j\omega})|^2 \cdot \Delta^2$$

$$W(z)\,W(z^{-1}) \cdot \Delta^2 = \Big|_{z = e^{j\omega}}$$

$$\Gamma(\omega) = \frac{|e^{j\omega} + \frac{1}{2}|^2 \cdot 9}{1} =$$

$$\Gamma(\omega) = \left| \cos\omega + j\,\mathrm{sen}\,\omega + \frac{1}{2} \right|^2 \cdot 4$$

$$= \ldots$$

Es2 processo stoc. debolmente

$$v(t) = y(t) + 2y(t-1) + y(t-2)$$

$$y(\cdot) \sim WN(0,1)$$

$W(z) = ?$ $\quad \Gamma_v(\omega) = ?$

$MA(2)$

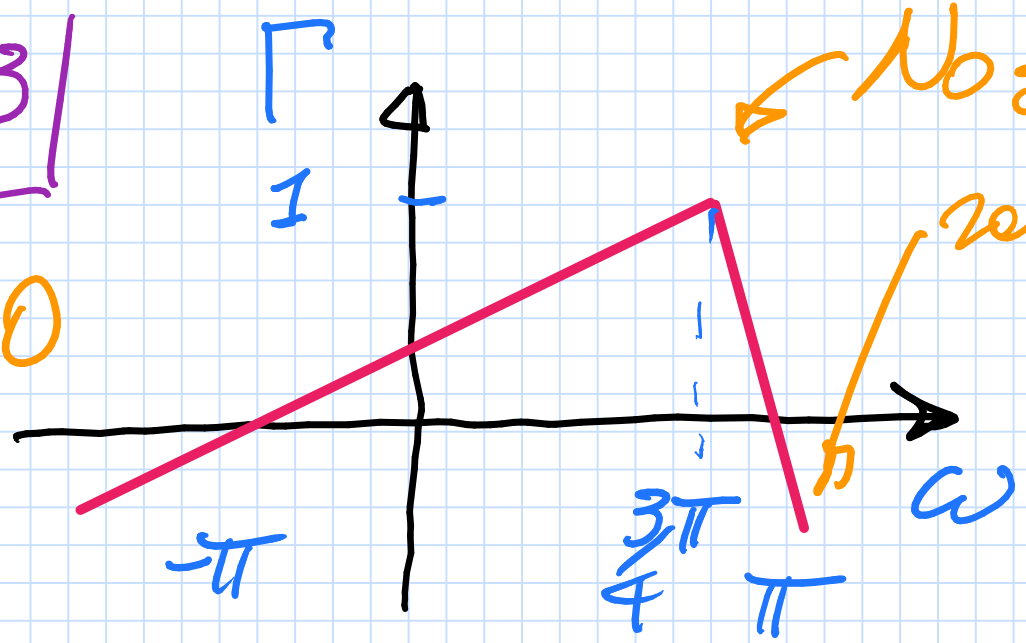
$$\gamma(\tau) = \begin{cases} 6 & \tau = 0 \\ 4 & \tau = \pm 1 \\ 1 & \tau = \pm 2 \\ 0 & |\tau| > 2 \end{cases}$$

$$W(z) = \frac{(z+1)^2}{z^2}$$

$$\Gamma(\omega) = |e^{j\omega} + 1|^4$$

Es. 3

NO



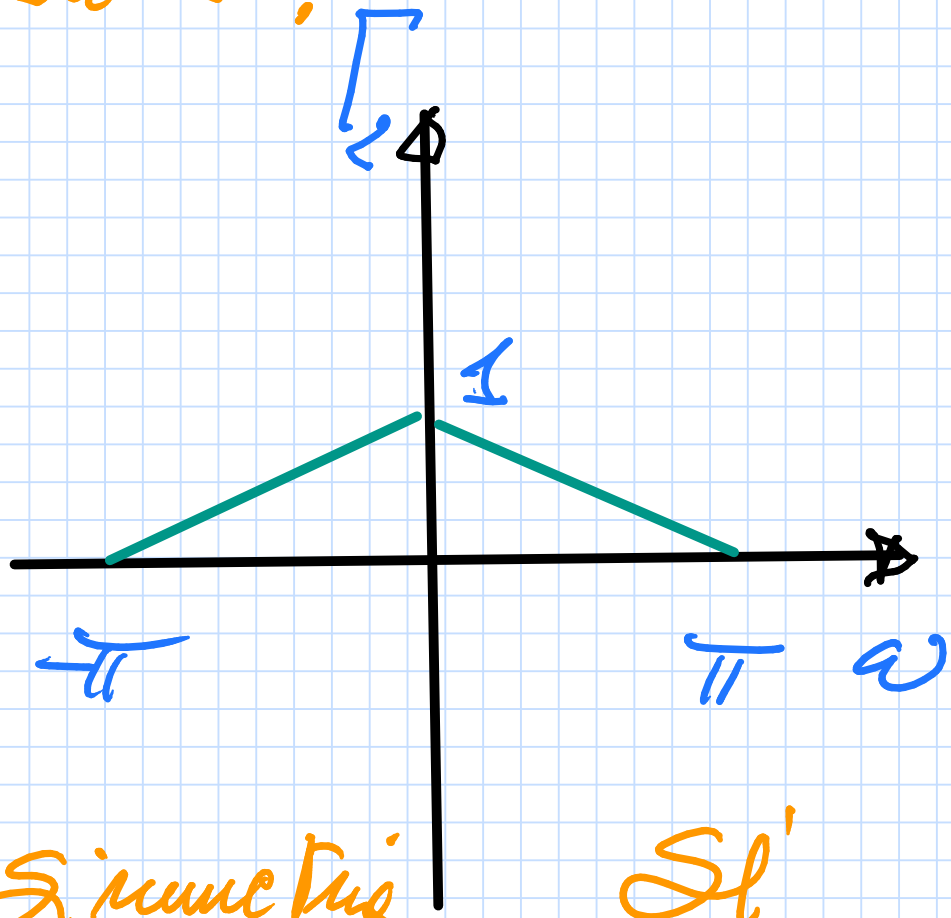
← Nonzero only when ω < 0!

Γ_1 ⟷ pass. bez.

? bez.

Γ_2

$\arg(v) = \gamma_2(0) = ?$



Zmienna zależna ω

Sł'

$$\rho(\tau) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Gamma(\omega) e^{i\omega\tau} d\omega$$

quindi

$$\rho(0) = \frac{1}{2\pi} \int_{-\pi}^{\pi} \Gamma(\omega) \cdot 1 \cdot d\omega$$

area sotto il grafico di Γ

$$\rho(0) = \frac{1}{2\pi} \cdot \frac{2\pi \cdot 1}{2} = 1/2$$

Es Si vuole stimare una costante c sulla base di osservazioni rumorose

$$d(t) = c + e(t) + \frac{1}{2} e(t-1)$$

$e(\cdot)$ è WWN$(0,1)$

(a) $\hat{c}(t) = \frac{1}{t} \sum_{1}^{t} \cdot \left[d(j) - \frac{3}{2} \right]$

(b) $\hat{c}(t) = \frac{1}{t} \sum_{1}^{t} \cdot d(j)$

$v(t) = c(t) + \frac{1}{2} c(t)$ $\quad c(\cdot) \sim U(1,1)$

$E[v] = ?$ $\quad E(v) = E[c(t)] + \frac{1}{2} E[c(t-1)]$

$= 1 + \frac{1}{2} = \frac{3}{2}$

(a) $2 \to c$

(b) $\tilde{c} \to c + \frac{3}{2}$

Consistenza di $\hat{c}$

$$\mathrm{var}\left[\hat{c}(t)\right] \xrightarrow[t \to +\infty]{} 0$$

$$\mathrm{var}\left[\hat{c}(t)\right] = E\left[\hat{c}(t) - c\right]^2$$

$$= E\left[c - \frac{1}{t}\sum_{i=1}^{t} \cdot \left(d(i) - \frac{3}{2}\right)\right]^2$$

$$=$$

$$= E\left\{\left[c - \frac{1}{t}\sum_{i}^{t}\cdot\left(c + e(t) + \frac{1}{2}e(t-1)\right) + \right.\right.$$

$$\left.\left. - \frac{3}{2}\right)\right]^2\Bigg\}$$

fuori da Σ_i

$$= E\left\{\left[\phi - \frac{1}{k}\cdot k\phi - \frac{1}{t}\sum_{i}\cdot\left(e(t) + \frac{1}{2}e(t-1)\right)\right.\right.$$

$$\left.\left. - \frac{3}{2}\right)\right]^2\Bigg\} =$$

$$= E\left\{\left[\frac{1}{t}\sum_{1}^{t}\cdot\left(-e(t)-\frac{1}{2}e(t-1)+\frac{3}{2}\right)\right]^2\right\}$$

definisco un processo MA(1):

$$v(t) = -e(t) - \frac{1}{2}e(t-1) + \frac{3}{2}$$

$e(\cdot) \sim WN(1,1)$ $\quad E(v) = 0$

$v(\cdot) \Rightarrow MA(1)$

$v(\cdot)$ MA(1)

$$\gamma(0) = \left(1 + \frac{1}{4}\right)\cdot 1 = \frac{5}{4} \qquad \gamma(\pm 1) = \frac{1}{2}$$

$$\mathrm{var}\left[\hat{c}(t)\right] = E\{\cdots\}^2$$

$$= E\left\{ \frac{1}{t^2} \left[\sum_{1}^{t} \cdot\, v(i) \right]^2 \right\}$$

$$= \frac{1}{t^2} E \left\{ \left[\sum_{1}^{t} v(i) \right]^2 \right\}$$

$$= \frac{1}{t^2} E \left\{ v^2(1) + v^2(2) + \cdots v^2(t) + \right. \Rightarrow$$
$$\left. 2v(1)v(2) + 2v(2)v(3) + \cdots \right\}$$

$$\left[v(1) + v(2) + v(3) \right]^2 = v^2(1) + v^2(2) +$$
$$+ v^2(3) + 2v_1v_2 + 2v_2v_3 + 2v_1v_3$$

$$= \frac{1}{t^2} \left\{ \sum_{1}^{t} E[v^2(i)] \right.$$

$$\left. + 2 \sum_{i=1}^{t-1} E[v(i)\, v(i+1)] \right\}$$

$$= \frac{1}{t^2} \left\{ \sum_{1}^{t} \gamma(0) + 2 \sum_{1}^{t-1} \gamma(1) \right\} =$$

$$\Rightarrow \operatorname{var}\hat{c}(t) = \frac{1}{t^2}\left\{t \cdot \frac{5}{2} + 2 \cdot \frac{1}{2} \cdot (t-1)\right\}$$

② /

$$= \frac{6t-1}{4t^2} \longrightarrow 0 \qquad t \to \infty$$

Es 2 grandezze x, y legate da relazione

$$y = f(x)$$

x	5	8	2	-1	-4	3
y	7	7	4	0	-6	3

Supponiamo che ci sia legame lineare

$$y = ax \qquad a \in \mathbb{R}$$

Stima dei parametri del modello

$$\hat{a} \longrightarrow J(a) = \frac{1}{N} \sum_{i=1}^{N} \left(y_i - a x_i \right)^2$$

modello $y = a x + \varepsilon$

$\varepsilon \sim WN(0, \sigma^2)$

Soluzione stima $\longrightarrow$ quella che minimizza J!

Słownie

$$\hat{\alpha} = \left[\sum_{i=1}^{n} x_i^2\right]^{-1} \cdot \left[\sum_{i=1}^{n} y_i \cdot x_i\right]$$

$$\hat{\alpha} \approx 1{,}1$$

$$y = 1{,}1 \cdot x$$

E se il modello fosse $y = ax + b$?

Se funzionale J diventa

$$J(\theta) = \frac{1}{N} \sum_{1}^{N} i \left(y_i - [x_i \; 1] \begin{bmatrix} a \\ b \end{bmatrix} \right)^2$$

$$\theta = \begin{bmatrix} a \\ b \end{bmatrix} \qquad \left(y_i - \varphi_i^T \theta \right)^2$$

Risolvo:

$$\begin{bmatrix} a \\ b \end{bmatrix} = \left(\sum_{i}^{n} \cdot \begin{bmatrix} x_i^2 & x_i \\ x_i & 1 \end{bmatrix} \right)^{-1} \cdot$$

$$\left(\sum_{i}^{n} \cdot \begin{bmatrix} x_i \cdot \\ 1 \end{bmatrix} \cdot y_i \right)$$

$$\begin{bmatrix} \hat{a} \\ \hat{b} \end{bmatrix} \longrightarrow \begin{bmatrix} 1{,}0954 \\ 0{,}1266 \end{bmatrix} \quad /\!/$$

E se il modello fosse $y = ax^2$

$$J(a) = \frac{1}{N} \sum_{i=1}^{N} \left(y_i - a x_i^2 \right)^2$$

$$\hat{R} = \frac{\sum_i y_i \cdot x_i^2}{\sum_i x_i^4} \pm 0,1123$$

3 modelli diversi

⇓

3 elettrodi diversi

Quale è il modello migliore?

Quali descrittori?

Es processo stazionario AR(1)

$$y(t) = \frac{1}{4} y(t-2) + e(t) - 2e(t-1)$$

$$e \sim WN(0,1)$$

	1	2	3	4	5	6	7
y	$5/2$	$7/5$	-1	$-7/5$	$-3/10$	$-4/5$	?

Determinare $\hat{y}(7/6)$ e $\hat{y}(7/5)$

$$E[y(0)] = 0$$

$$\hat{y}(7|6) \;??\; \hat{y}(7|5)$$

$$W(z) = \frac{1 - 2z^{-1}}{1 - \frac{1}{4}z^{-2}}$$

non è una forma canonica!

$$\hat{W}(z) = \frac{1-2z^{-1}}{\left(1-\frac{1}{2}z^{-1}\right)\left(1+\frac{1}{2}z^{-1}\right)} \cdot \frac{\left(1-\frac{1}{2}z^{-1}\right)}{\left(1-2z^{-1}\right)}$$

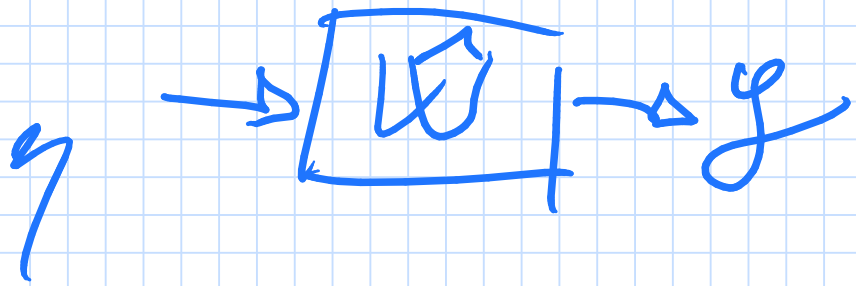
$$= \frac{1}{1+\frac{1}{2}z^{-1}}$$

$$W(z) \cdot W(z^{-1}) \cdot I = \hat{W}(z)\,\hat{W}(z^{-1}) \cdot I^2$$

$$I^2 = 4 \;\#$$

$$\hat{W}(z) = \frac{1}{1 + \frac{1}{2} z^{-1}}$$

$$\eta(\cdot) \sim WN(0,4)$$



Un forma canonica c'è un AR(1)

$$\begin{array}{r|l} 1 & 1+\frac{1}{2}z^{-1} \\ \hline -1-\frac{1}{2}z^{-1} & 1 \\ \hline -\frac{1}{2}z^{-1} & \end{array}$$

predittore ad 1 passo

$$\hat{W}(z) = 1 + z^{-1} \cdot \frac{-\frac{1}{2}}{1+\frac{1}{2}z^{-1}}$$

predittore

predittore eliminando del dato

$$\hat{y}(t+1/t) = -\frac{1}{2}y(t)$$

predittore ad 1 passo

$$\hat{y}(7|6) = -\frac{1}{2} y(6) = \frac{2}{5}$$

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predittore a 2 passi?

$$\begin{array}{r|l}
1 & 1+\frac{1}{2}z^{-1} \\
\hline
-1-\frac{1}{2}z^{-1} & 1-\frac{1}{2}z^{-1} \\
\hline
-\frac{1}{2}z^{-1} & \\
+\frac{1}{2}z^{-1}+\frac{1}{4}z^{-2} &
\end{array}$$

$$\hat{W}(z) = 1 - \frac{1}{2} z^{-1} + z^{-2} \cdot \frac{\frac{1}{4}}{1 + \frac{1}{2} z^{-1}}$$

$$W_2(z) = \frac{1}{4}$$

pred. e2 pesi
elim. de ricorsione

$$\hat{y}(7|5) = \frac{1}{4}\left(-\frac{3}{10}\right) = -\frac{3}{40} \;//$$