Trasformazione bilineare + criterio di R. H.

$p(z) = 0$

$$z = \frac{w+1}{w-1}$$

$w \in \mathbb{C}$

trasformazione bilineare

$$P(z) = z^2 - z - 1$$

$$P(z) = 0 \rightarrow$$

$$z_1 = \frac{1-\sqrt{5}}{2} \simeq -0{,}618$$

$$z_2 = \frac{1+\sqrt{5}}{2} \simeq 1{,}618$$

$$z^2 - z - 1 = 0 \Leftarrow z = \frac{w+1}{w-1}$$

$$\left(\frac{\omega+1}{\omega-1}\right)^2 - \left(\frac{\omega+1}{\omega-1}\right) - 1 = 0 \quad / \, (\omega-1)^2$$

$$\omega \neq 1$$

$$(\omega+1)^2 - (\omega+1)(\omega-1) - (\omega-1)^2 = 0$$

$$-\omega^2 + 4\omega + 1 = 0$$

$+w^2 - 4w - 1 = 0$

$q(w) = 0$

2	$+1$	-1
1	-4	
0	-1	

$1v + 1p$

trasf. bl.

$\Downarrow$

$\left.\begin{array}{l} 1 \text{ radice } Re(w) > 0 \\ 1 \text{ radice } Re(w) < 0 \end{array}\right\} q(w)$

1 radice $|\bar{z}| > 1$

1 radice $|\bar{z}| < 1$ $\quad$ per $P(z)$ INST.

$\boxed{1^\circ \text{ caso particolare}}$ $\exists\ \tilde{z} = -1$

$$p(-1) = 0$$

$$p(z) = (z+1)^2(z+10)$$

$$\Downarrow \quad z = \frac{w+1}{w-1}$$

$$\left(\frac{w+1}{w-1}+1\right)^2\left(\frac{w+1}{w-1}+10\right) = 0 \quad \Big/ (w-1)^3$$

$q(w)=0$ $\qquad 9w^2\underbrace{(11w-9)}=0$

$p(z)$ radice doppia in $z=-1$

$q(w)$ radice doppia in $w=0$

$w=9/11$

>0 !

$$4w^2(11w-9) = 44w^3 - 36w^2$$

3	44	0
2	-36	0
1	0	
0		

$\boxed{2^\circ \text{ caso particolare} \quad p(z) \text{ radice } z = +1}$

$$p(z) = (z-1)^2\left(z+\frac{1}{2}\right)$$

$$z = \frac{w+1}{w-1}$$

$$\left(\frac{w+1}{w-1} - 1\right)^2\left(\frac{w+1}{w-1} + \frac{1}{2}\right) = 0 \Big/ (w-1)^3$$

$q(3w+1)=0$

$q(w)=0$

$\neq 1 \; (3-2$

$p(z) \neq 3$

$z=+1$ radice doppia $\Longrightarrow$

$$q(3w+1) = g(w)$$

$$P(z)$$

$\mathrm{Re}(\ ) < 0$

$\#q = 3 - 2$

$\Rightarrow$

- radice $|\ | < 1$
- 2 radici in $z = +1$

Stabilità al variare di un parametro

$$
\begin{cases}
x_1(k+1) = 2x_1(k) - 2x_2(k) + u(k) \\
x_2(k+1) = 2x_1(k) - \gamma x_2(k) \\
x_3(k+1) = -2x_1(k) + \gamma x_2(k) \\
y(k) = x_3(k)
\end{cases}
\qquad \gamma \in \mathbb{R}
$$

Come varia la stabilità al variare di γ?

$$p(z) = \det[zI - A]$$

$$\begin{vmatrix} z-2 & 2 & 0 \\ -2 & z+\gamma & 0 \\ 2 & -\gamma & z \end{vmatrix} = z\left[z^2 + (\gamma-2)z + 2(2-\gamma)\right]$$

sviluppo secondo la 3ª colonna

$$z^2 + (\nu - 2)z + 2(2 - \nu) = 0$$

$$z = \frac{w+1}{w-1}$$

$$\left(\frac{w+1}{w-1}\right)^2 + (\nu - 2)\left(\frac{w+1}{w-1}\right) + 2(2 - \nu) = 0 \quad / \, (w-1)^2$$

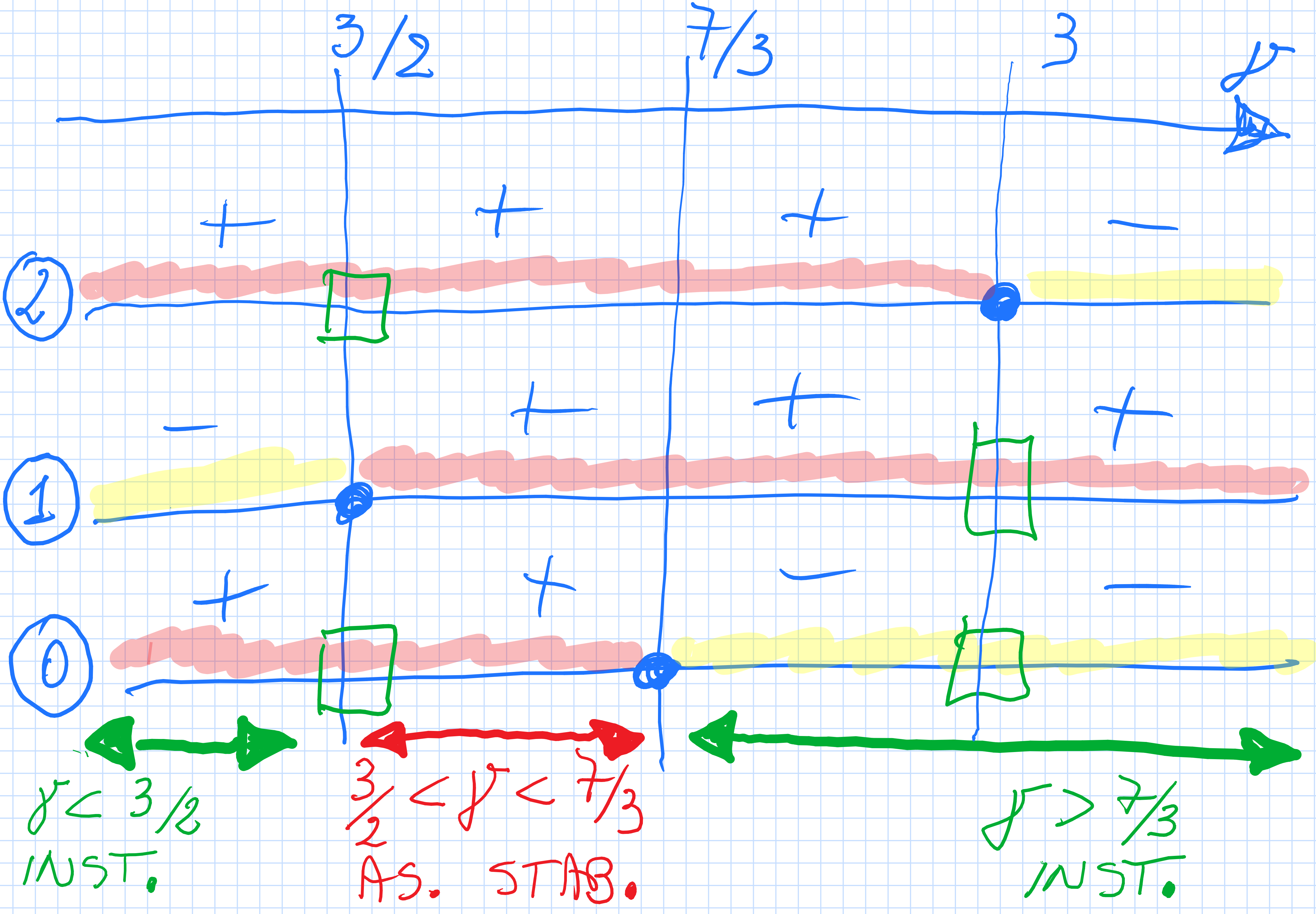
$$(3 - \nu)w^2 + 2(2\nu - 3)w + (7 - 3\nu) = 0$$

2	$3-y$	$(7-3y)$
1	$2(2y-3)$	
0	$(7-3y)$	

$$3 - y \geq 0 \Rightarrow y \leq 3$$

$$2y - 3 \geq 0 \Rightarrow y \geq 3/2$$

$$7 - 3y \geq 0 \Rightarrow y \leq 7/3$$



3/2
7/3
3
γ
+
+
+
−
2
−
+
+
+
1
+
+
−
−
0
γ < 3/2
INST.
3/2 < γ < 7/3
AS. STAB.
γ > 7/3
INST.

$\boxed{k = 3/2} \longrightarrow$ 2 radici immag. pure per $q(w)$

$\Downarrow$

2 radici con $Re(\,) = 0$ in w

$\Downarrow$

2 radici con $|\cdot| = 1$ in z

STAB. SEMPL.

(sulla circonferenza di raggio 1 e centro l'origine)

$\boxed{k = 7/3}$ $\longrightarrow$ radice in $w = 0$

$\Downarrow$

radice in $z = -1$ per $p(z)$

$\Downarrow$ in $|\cdot| < 1$

Semplificabile

$\boxed{k = 3}$ $q(w)$ ha grado 1, radice $\mathrm{Re} > 0$

$\Downarrow$ $p(z)$ ha radice in $z = +1$ INST.

ha radice $|\cdot| > 1$