

$$L(s) = \frac{1}{2} \frac{(1-s)(1+10s)}{(1+s)^2}$$

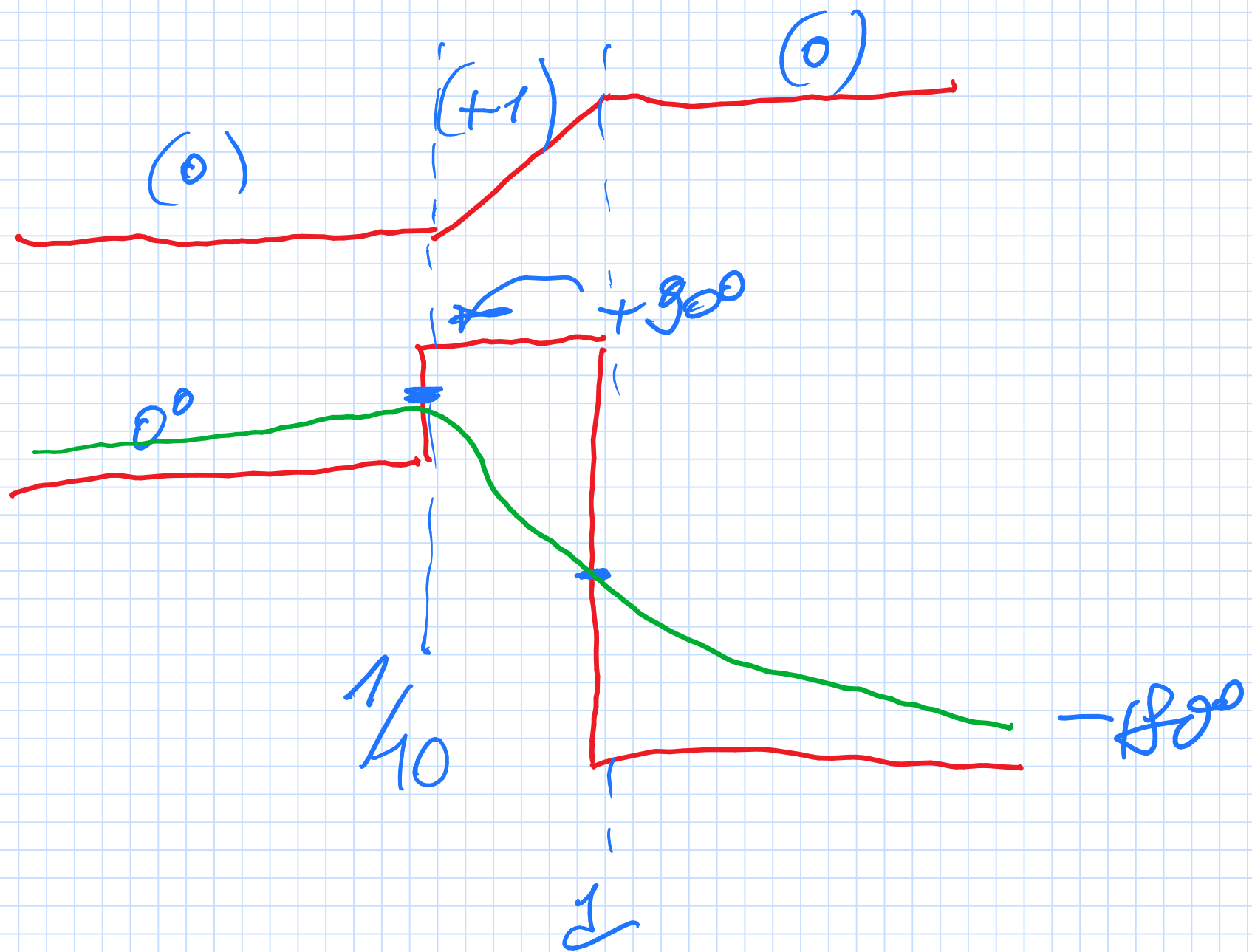
Esercizio: tracciamento
di diagrammi di
Bode e Nyquist

$$\mu_s = \frac{1}{2}$$

$$z_1 = -\frac{1}{10} \longleftrightarrow \omega_{z_1} = \frac{1}{10}$$

$$z_2 = +1 \longleftrightarrow \omega_{z_2} = 1$$

$$p_1 = -1 \longleftrightarrow \omega_{p_1, p_2} = 1$$

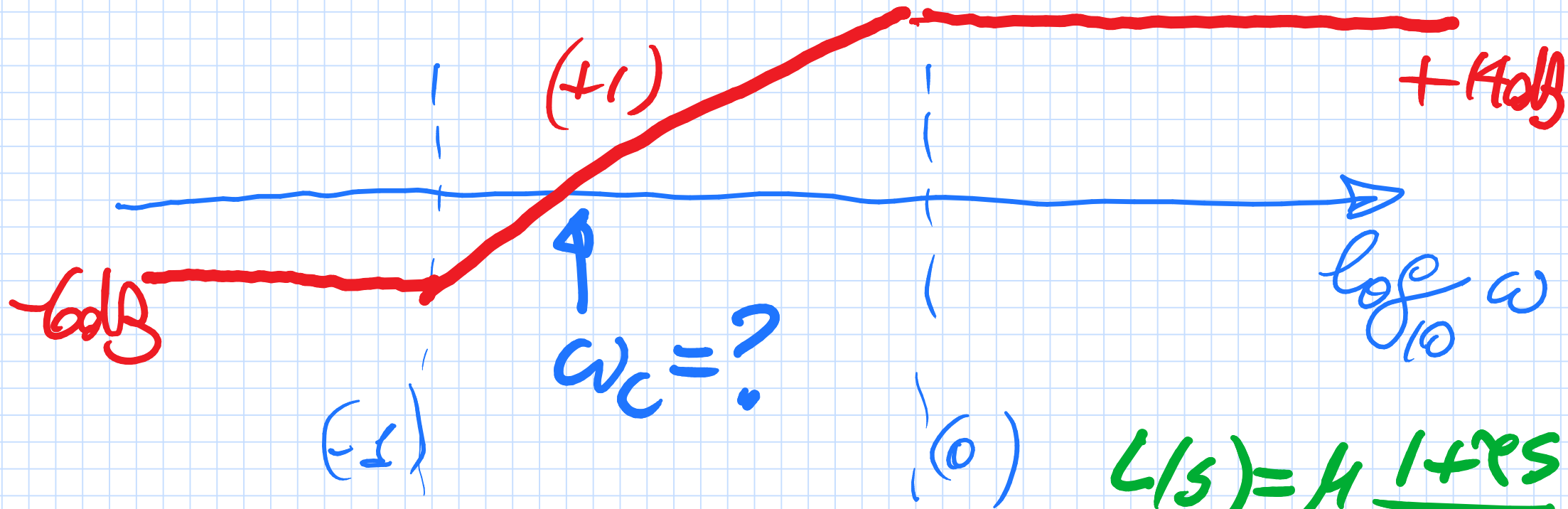


$$L(j\omega) = \frac{1}{2} \frac{(1-j\omega)(1+10j\omega)}{(1+j\omega)^2}$$

$$0 \leq \omega < \frac{1}{10}$$

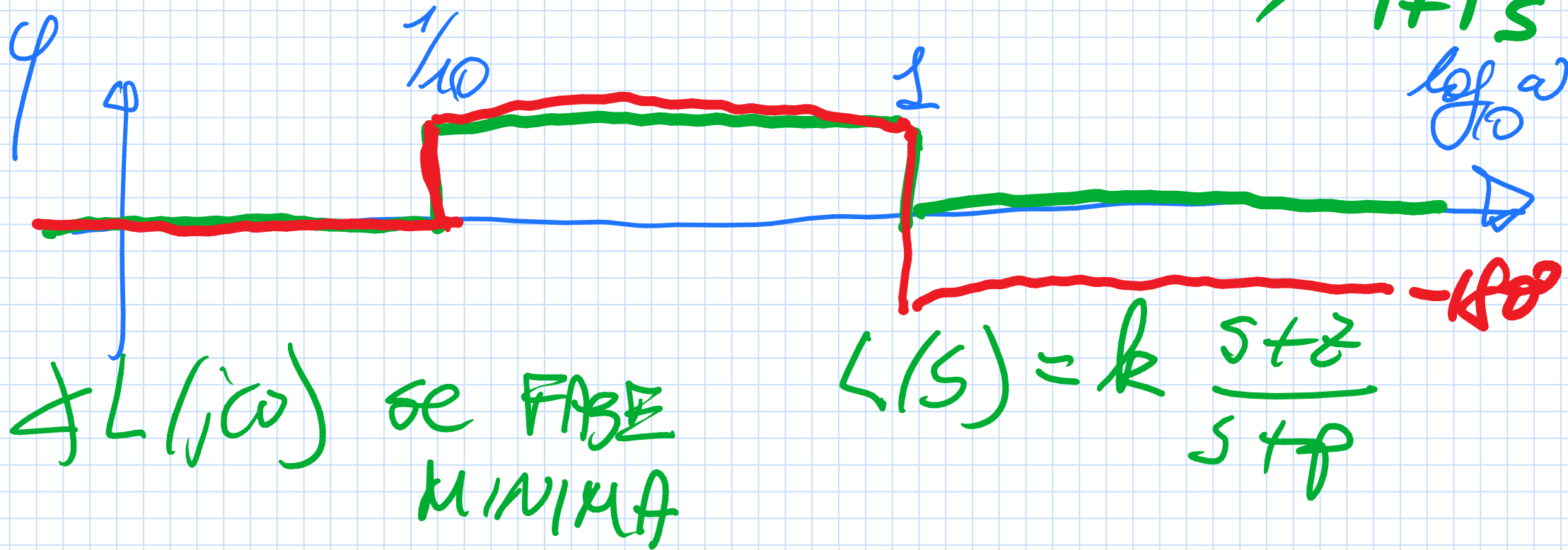
$$|L(j\omega)| = \frac{1}{2} \cdot 1 \cdot 1 / 1 \cdot 1$$

$$|L|_{dB} = 20 \log_{10} \frac{1}{2} = -20 \log_{10} 2 = -6 \text{ dB}$$



$$L(s) = K \frac{1 + \tau s}{1 + T s}$$

$\log_{10} \omega$



$$L(s) = K \frac{s + z}{s + p}$$

-180°

$\angle L(j\omega)$ or FASE MINIMA

$$L(s) = \frac{1}{2} \frac{(1-s)(1+10s)}{(1+s)^2}$$

$$\frac{1}{10} \leq \omega < 1$$

$$\tilde{L}(s) = \frac{1}{2} \cdot \frac{1-10s}{1^2}$$

$$|L(j\omega)|_{dB} = -6 + 20 \log_{10} (10\omega) - 0$$

$$= -6 + 20 + 20 \log_{10} \omega$$

$$= +14 + 20 \log_{10} \omega$$

espressione
del tratto
di generale
del modulo
per $0.1 \leq \omega < 1$

$$|L|_{dB} = +14 + 20 \log_{10} \omega$$

$$\omega_c \rightarrow |L|_{dB} = 0 \quad -14 = 20 \log_{10} \omega$$

$$\log_{10} \omega = -\frac{7}{10}$$

$$\omega_c = 10^{-7/10} \approx 0,19 \text{ rad/s}$$

$$|L(j\omega)| = 1$$

$$\frac{1}{2} \frac{|1-j\omega| \cdot |1+j10\omega|}{|(1+j\omega)^2|} = 1$$

$$|L(j\omega)| = 1 \quad \frac{1}{2} \frac{|1 + 10\omega^2 + 9j\omega|}{|1 - \omega^2 + 2j\omega|} = 1$$

$$\frac{(1 + 10\omega^2)^2 + 81\omega^2}{(1 - \omega^2)^2 + 4\omega^2} = 4$$

$$96\omega^4 + 93\omega^2 - 3 = 0$$

$$\omega^2 \triangleq t$$

$$96t^2 + 93t - 3 = 0$$

$$t = \frac{-1}{\dots}$$

$$\underline{20,03125} \rightarrow \omega_c \underline{\underline{20,177 \text{ rad/s}}}$$

$$\begin{aligned}
 \angle L(j\omega_c) &= \angle \frac{1}{2} + \angle (1 - j\omega_c) + \\
 &\quad + \angle (1 + j10\omega_c) + \\
 &\quad - 2\angle (1 + j\omega_c)
 \end{aligned}$$

$$\begin{aligned}
 &0^\circ + \cancel{\arg(-\omega_c)} \\
 &| - \arg(\omega_c) + \arg(10\omega_c) - 2\arg \omega_c \\
 &= -3\arg \omega_c + \arg(10\omega_c) \Rightarrow
 \end{aligned}$$

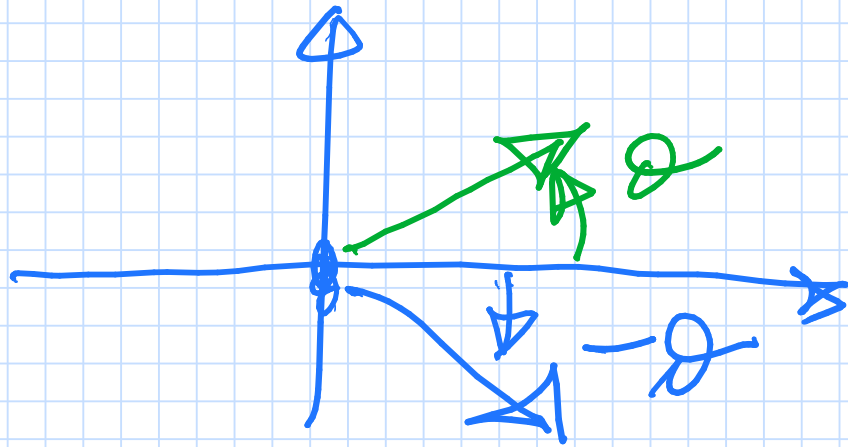


grafico "a 4 quadranti"
per determinare il
contributo del termine
 $\angle(\dots)$ in termini di
arg $\angle(\dots)$

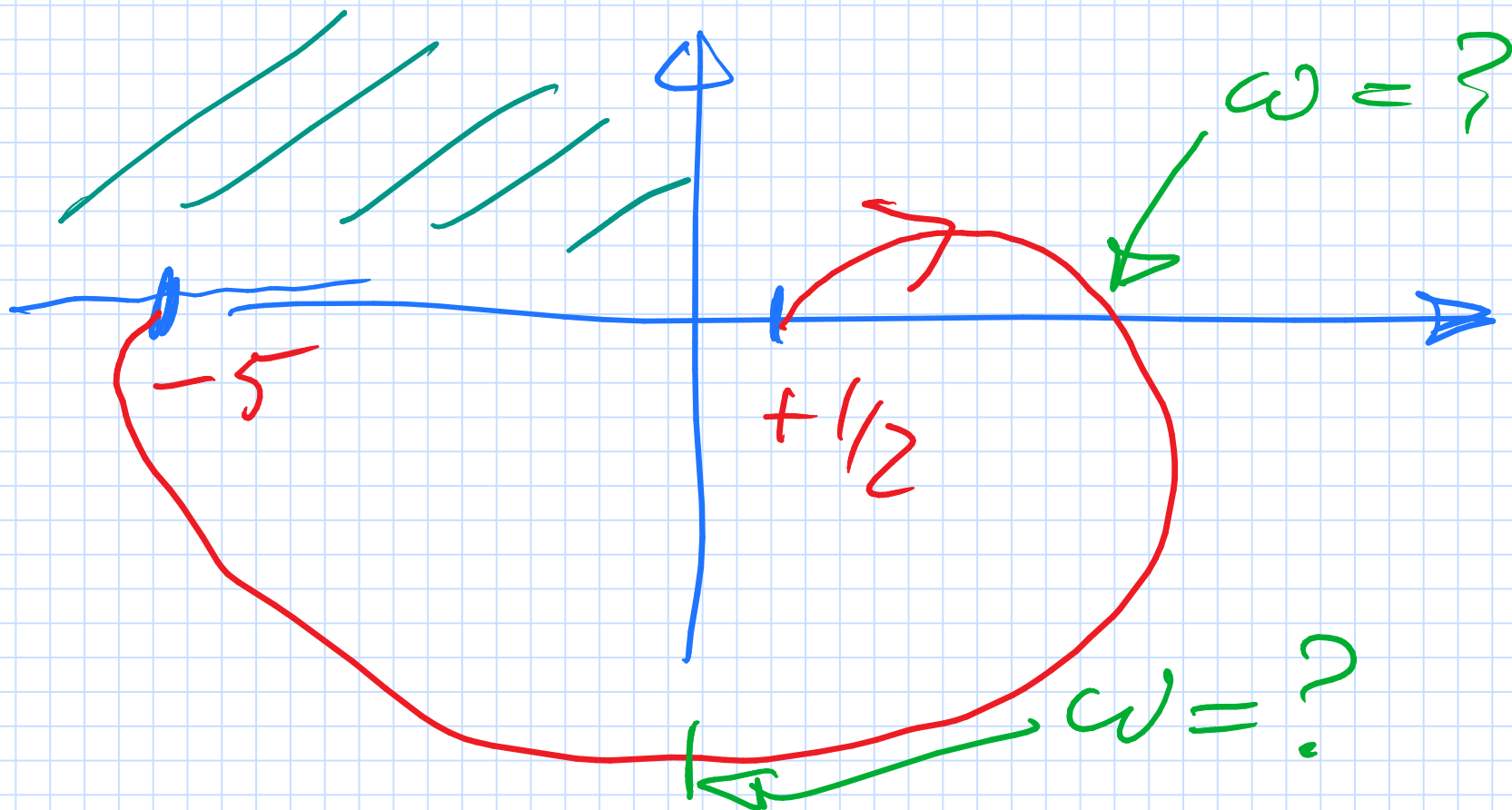
In definitiva risulta:

$$\angle L(j\omega_c) \geq +20^\circ$$

$$\omega = 0 \quad | \cdot | = \frac{1}{2} \quad \angle = 0^\circ$$

diagramma polare
di $L(j\omega)$

$$\omega \rightarrow \infty \quad | \cdot | \rightarrow 5 \quad \angle \rightarrow -180^\circ$$



$$R = \frac{1}{2} \frac{(1+10\omega^2)(1-\omega^2) + 16\omega^2}{(1-\omega^2)^2 + 4\omega^2}$$

$$I = \frac{1}{2} \frac{+9\omega(1-\omega^2) - 2\omega(1+10\omega^2)}{(1-\omega^2)^2 + 4\omega^2}$$

$\underline{t}=0 \rightarrow \omega=0$ 4 le corrente
fissa

$$29\omega^2 = +7$$

$$\rightarrow \bar{\omega} = \sqrt{\frac{7}{29}} \approx 0,49$$

$$R(\bar{\omega}) = 2,25$$

$$R=0 \quad (1+10\omega^2)(1-\omega^2) + 18\omega^2 = 0$$

✓

$$R=0$$

$$10\omega^4 - 27\omega^2 - 1 = 0$$

$$\omega^2 = t$$

$$10t^2 - 27t - 1 = 0$$

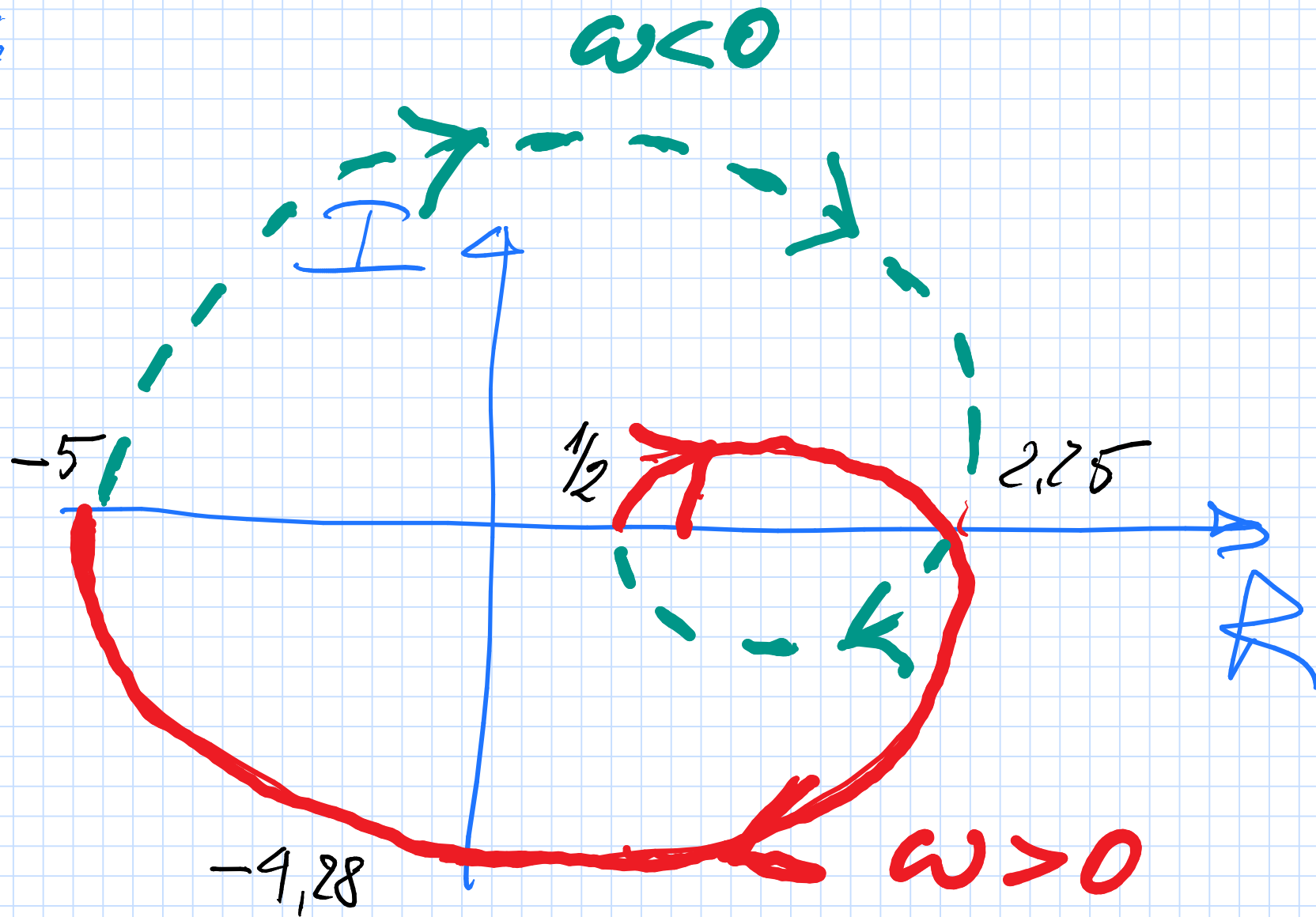
$$\begin{matrix} t \\ 1 \end{matrix} \quad \begin{matrix} 2,7365 \\ -0,03 \dots \end{matrix}$$

now see.!

$$\omega \approx 1,654$$

$$I(\omega) \approx -9,28 //$$

In definitiva il diagramma completo di Nyquist
è:

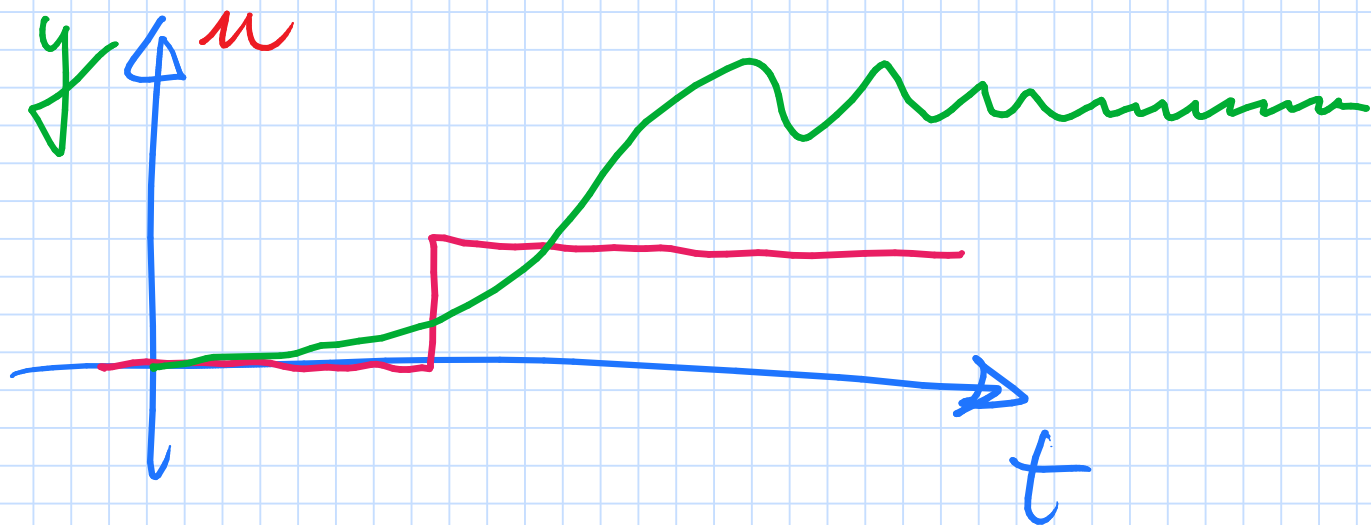


Esercizio

$$L(s) = \frac{4}{1 + \frac{s}{10}}$$

In quanto tempo la
risposta transitoria allo
scalino unitario si
è praticamente estinta?

E se fosse $L(s) = 4 \frac{1+s}{1+s/10}$?



Quanto vale
il tempo di
assettamento t_q
nei 2 casi?

Esercizio: Tracciamento
di diagrammi di Bode
e di Nyquist

$$L(s) = \frac{4(1+s)}{s(1+\frac{1}{10}s)}$$

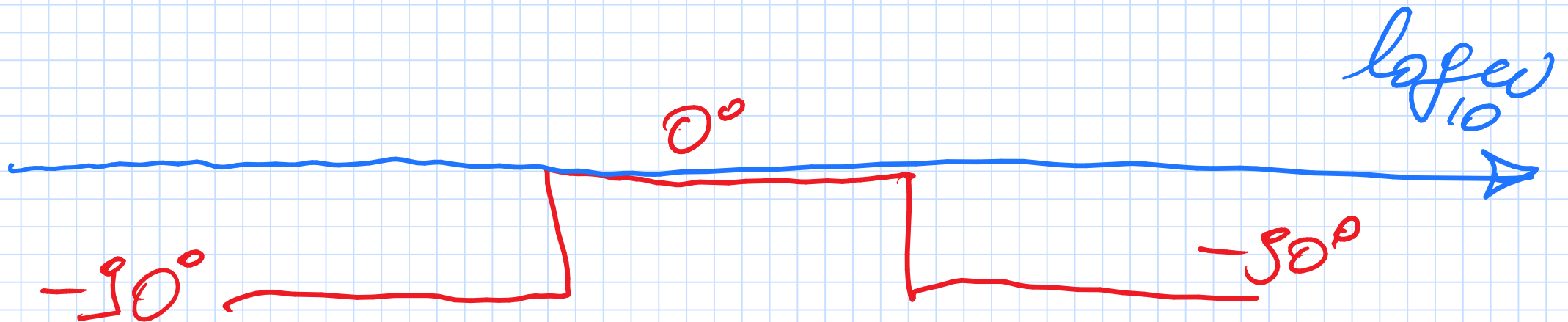
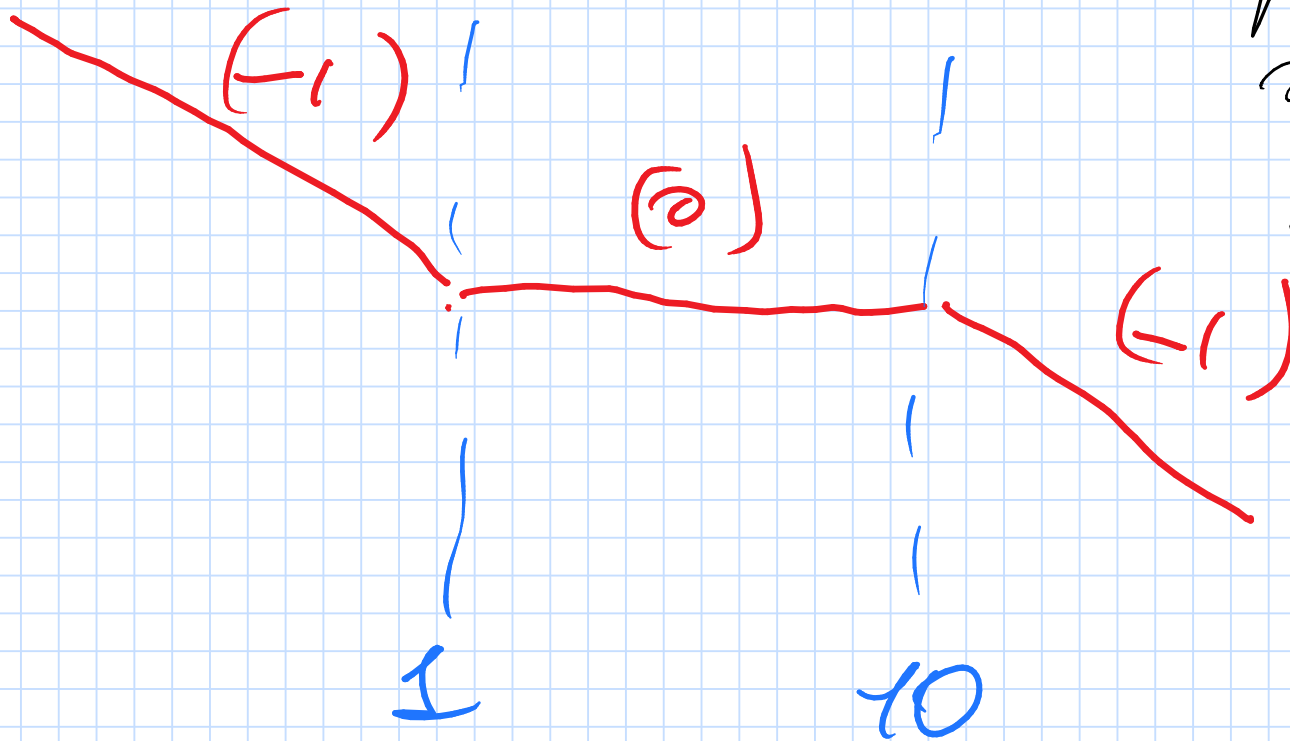
$$\mu = 4$$

$$z_1 = -1 \rightarrow \omega_{z_1} = +1$$

$$p_1 = 0 \rightarrow \omega_{p_1} = 0$$

$$p_2 = -10 \rightarrow \omega_{p_2} = 10$$

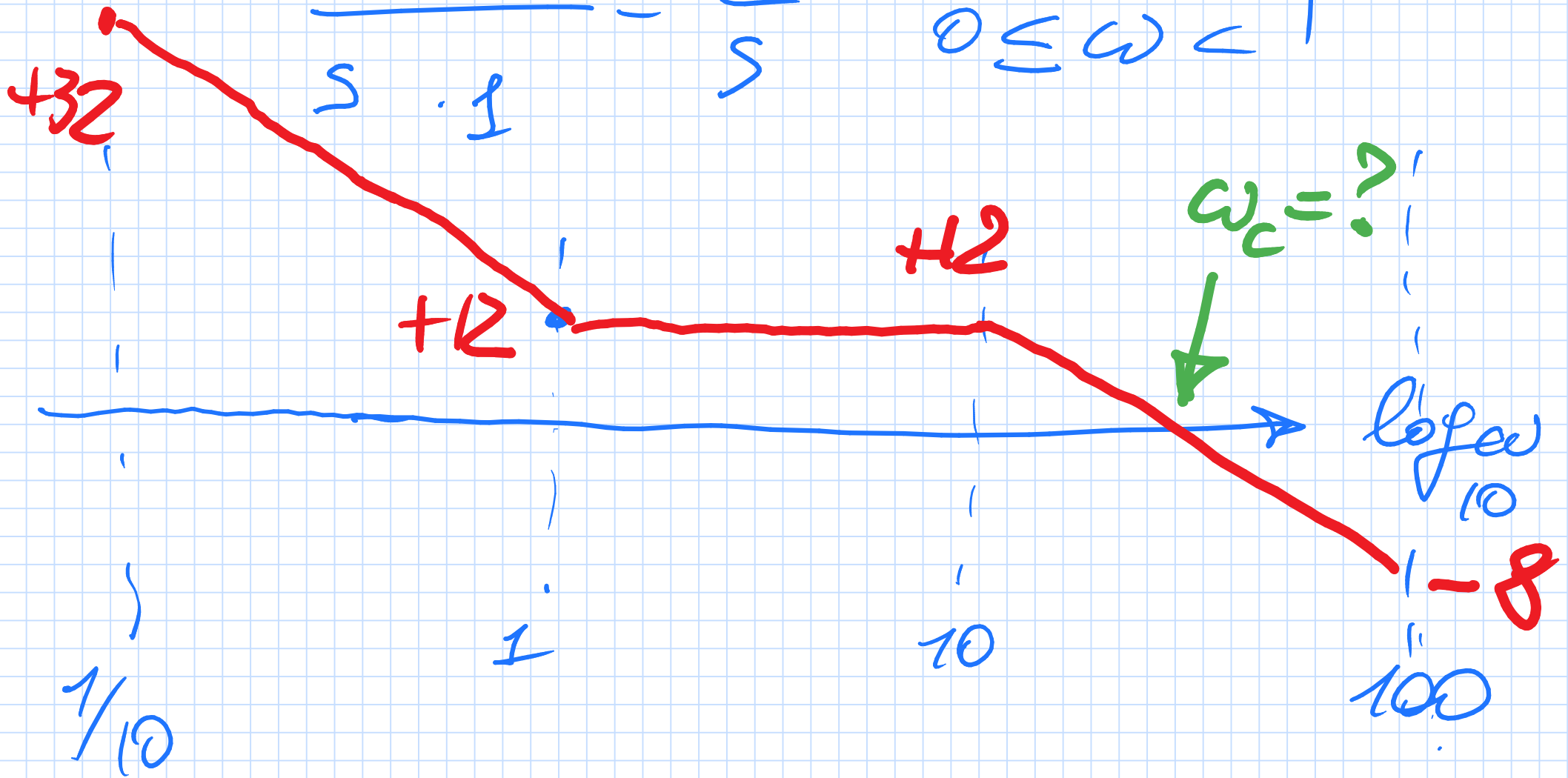
primus
schlus
qualitäts



$$L(s) = \frac{4 \cdot s}{s \cdot 1} = \frac{4}{s}$$

equazione del 1° tratto di
guadagno del modulo

$$0 \leq \omega < 1$$



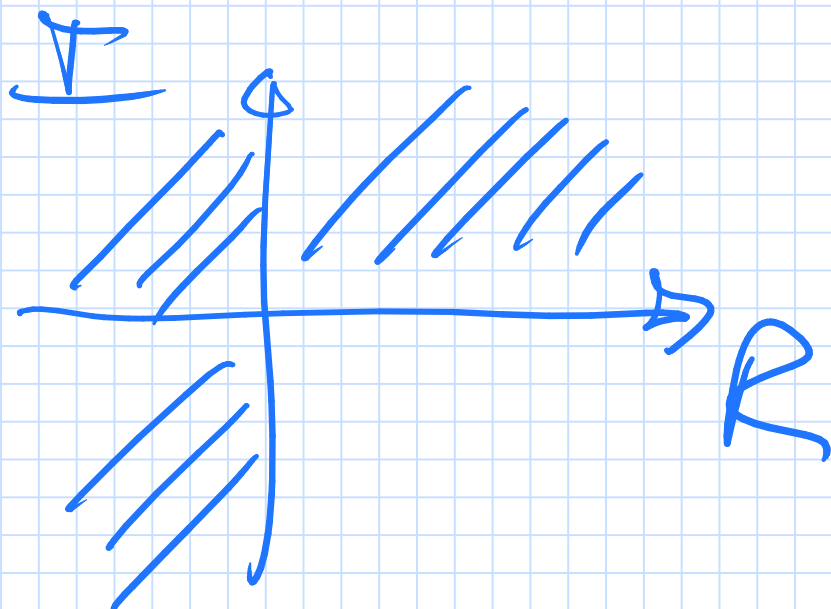
$$L(j\omega) = \frac{9(1+j\omega)}{j\omega(1+j\frac{\omega}{10})} =$$

phase margin phase

$$= \underbrace{9 \cdot \frac{1/10}{1 + \frac{\omega^2}{100}}}_{\text{amplitude}} \underbrace{- 4j \frac{1 + \omega^2/10}{\omega \left[1 + \frac{\omega^2}{100} \right]}}_{\text{phase}}$$

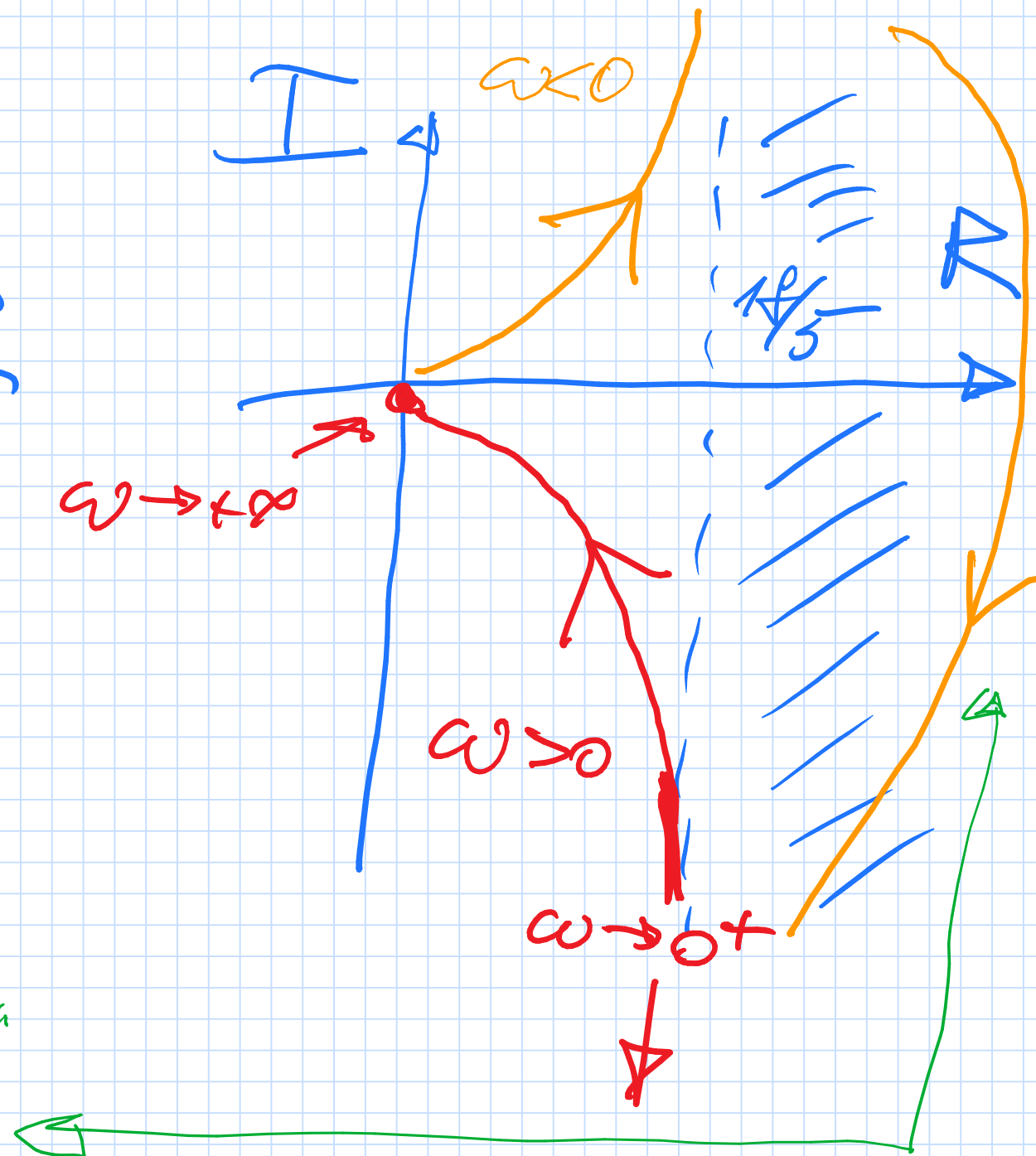
amplitude > 0
 $\omega \rightarrow 0^+ \Rightarrow R \rightarrow 18/5$

$\omega > 0 \Rightarrow I < 0$
 $\omega \rightarrow 0^+ \Rightarrow I \rightarrow -\infty$



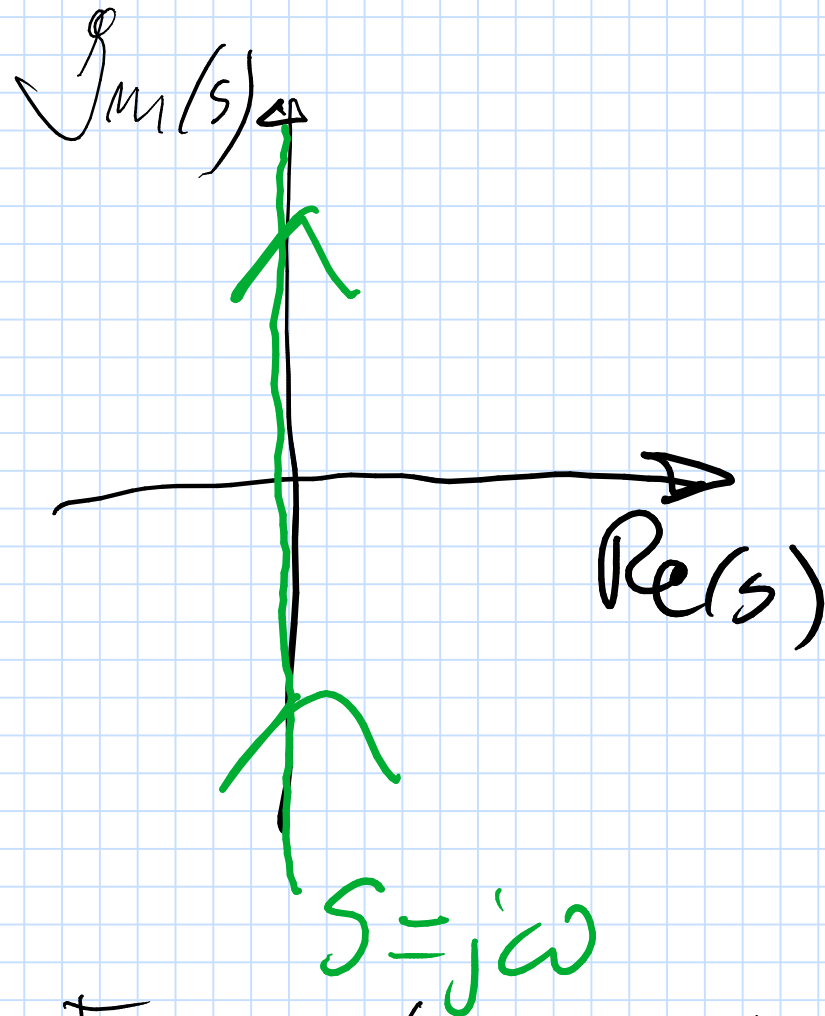
- dello studio del segno di $R(\omega)$ ed $I(\omega)$
- dei due termini di Bode (fase)

"divergenza all'infinito"

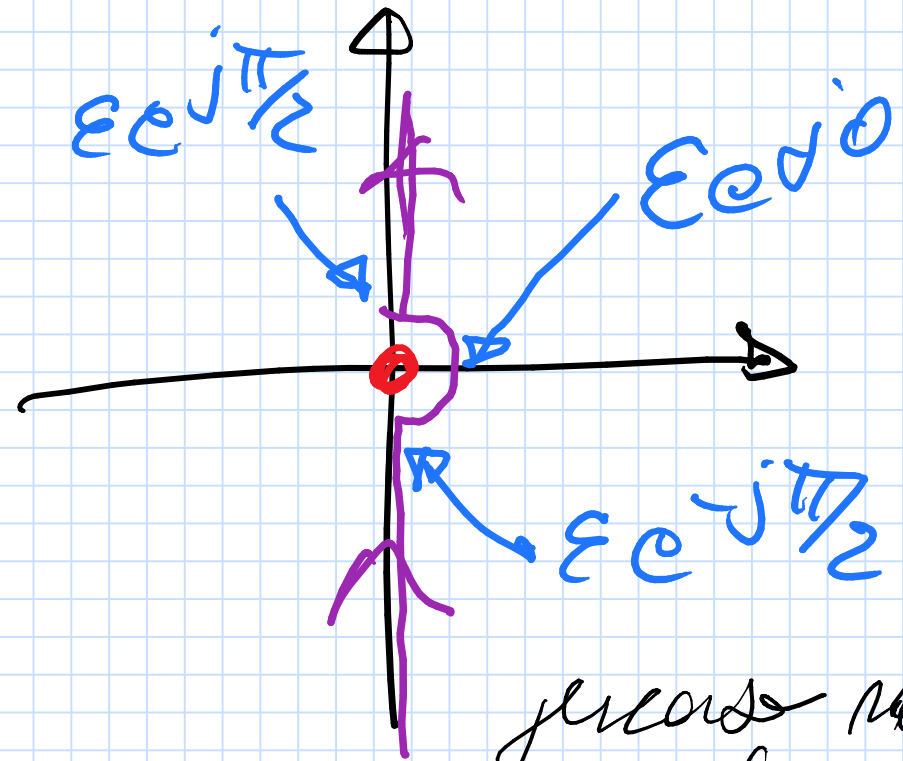


giusto frenare alla
"chiusura all' ∞ "

$$G(s) \Big|_{s=j\omega}$$



restituzione / recupero
se non ci sono poli su asse immaginaria.



prende nel caso
di polo in 0

$$G(s) = \frac{4(1+s)}{s(1+\frac{s}{10})}$$

← Sviluppo in serie di Laurent nell'intorno del polo in $s=0$

$$G(s) = \frac{C_{-1}}{s} + C_0 + C_1 s + \dots$$

$$= \frac{4}{s} + \frac{18}{5} + \dots$$

considero solo questi termini dello sviluppo

$$\tilde{G}(s) \approx \frac{4}{s} + \frac{18}{s}$$

considero i punti

$$a \rightarrow s_a = \varepsilon e^{j0}$$

$$b \rightarrow s_b = \varepsilon e^{j\pi/2}$$

$$c \rightarrow s_c = \varepsilon e^{-j\pi/2}$$

considero solo
i termini dello

sviluppo di Laurent

che corrispondono

ai termini dominanti

per $s \rightarrow 0$ di parte

reale ed immaginaria

Valuto $\tilde{G}(s)$ in corrispondenza
dei tre punti s_a, s_b, s_c

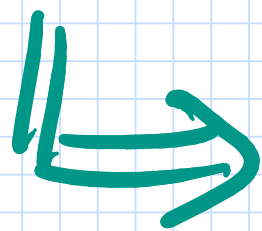
$$s = s_c = \varepsilon e^{-j\pi/2}$$

$$\hookrightarrow \tilde{G}(s_c)$$

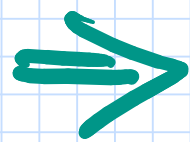


$$\frac{4}{5} e^{j\pi/2} + \frac{18}{5}$$

$$s = s_b = \varepsilon e^{j\pi/2}$$

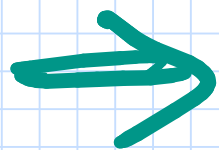


$$\tilde{G}(s_b)$$



$$\frac{4}{5} e^{j\pi/2} + \frac{18}{5}$$

$$s = s_a = \varepsilon e^{j0}$$

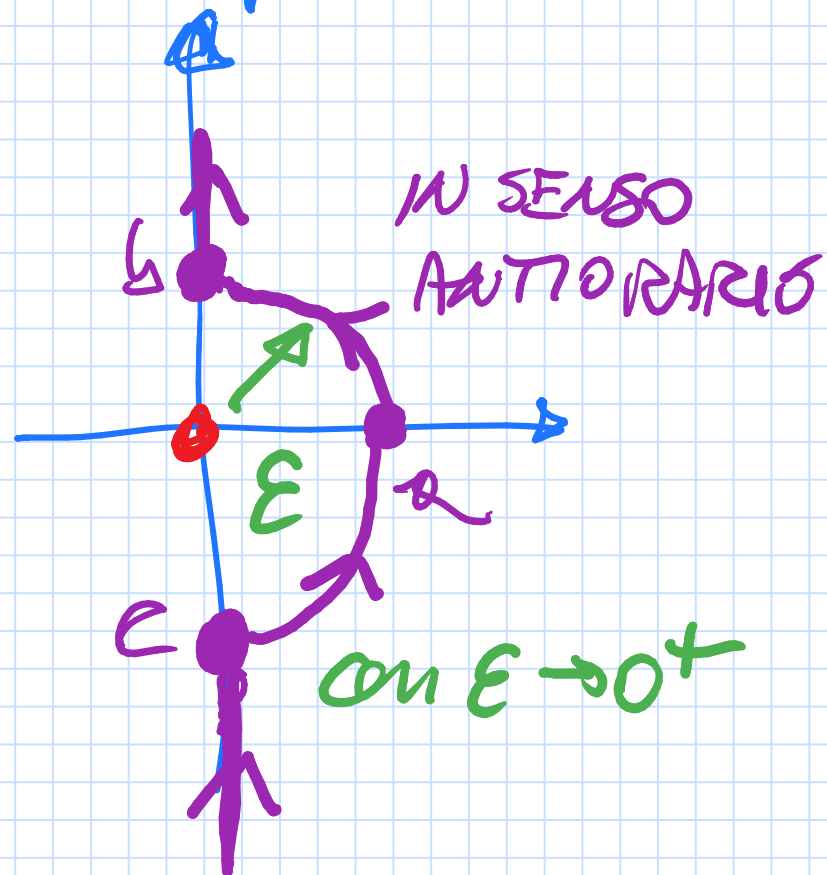


$$\tilde{G}(s_a)$$



$$\frac{4}{5} + \frac{18}{5}$$

percorso per risolvere $G(j\omega)$



aggirando il polo

seguendo il verso di percorrenza
da c ad a ed infine b , nel piano
della $L(j\omega)$ il diagramma "si divide" $\Rightarrow$

Nel diagramma pole
di $G(z)$ quici 3
già detto origine e

