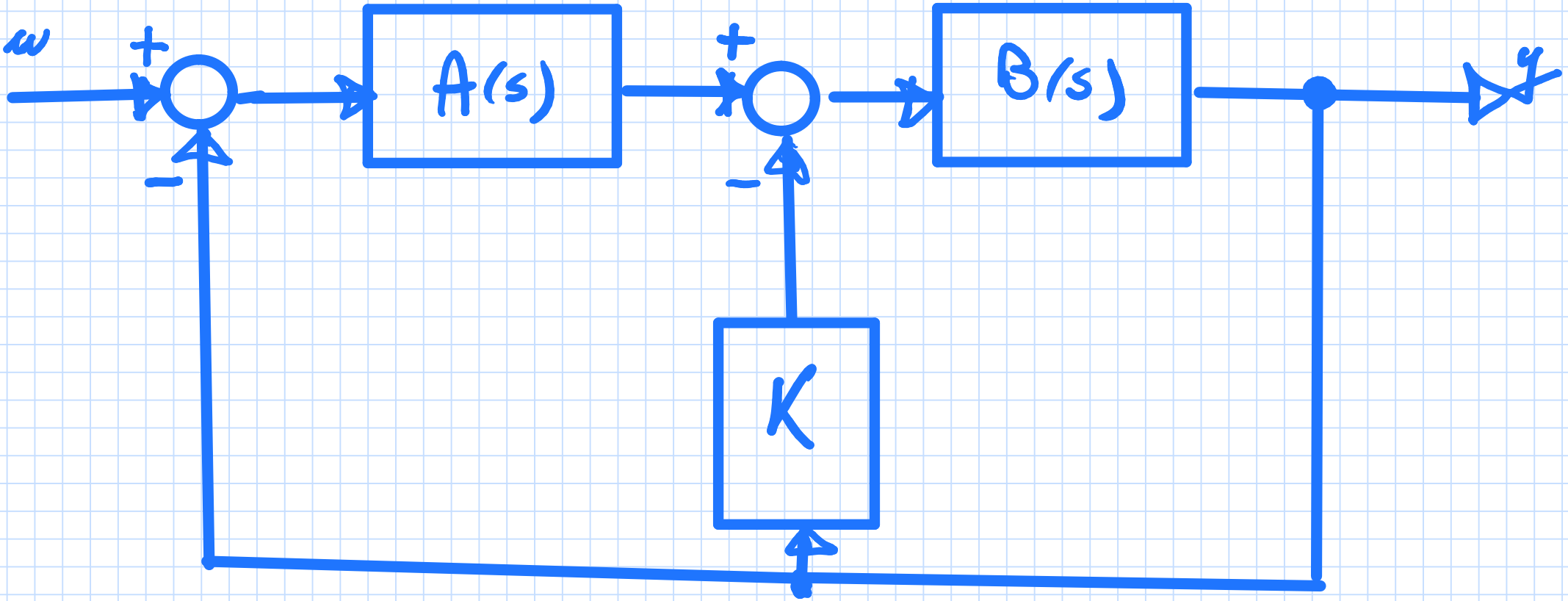
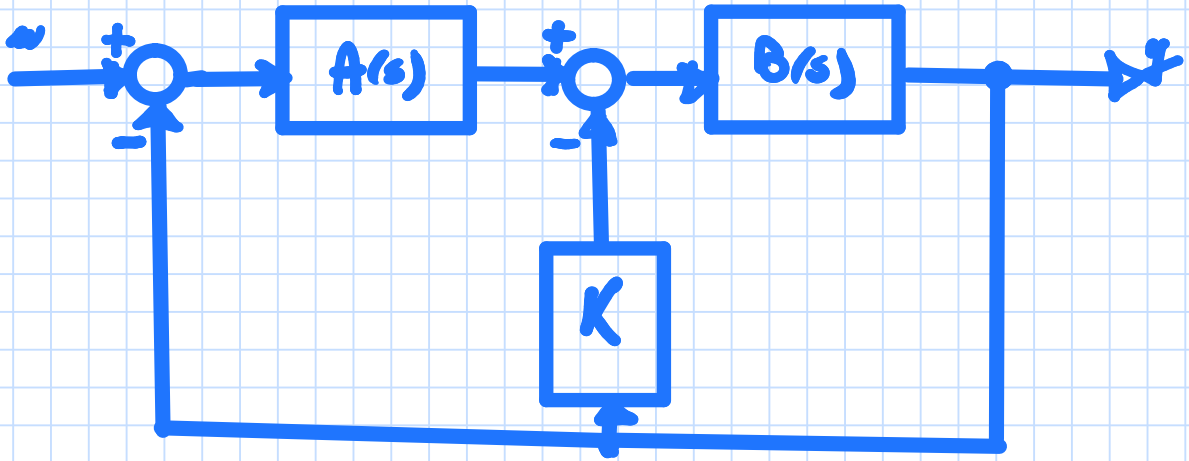
FONDAMENTI DI AUTOMATICA

Esercizi risolti: analisi di schemi a blocchi

1° esercizio

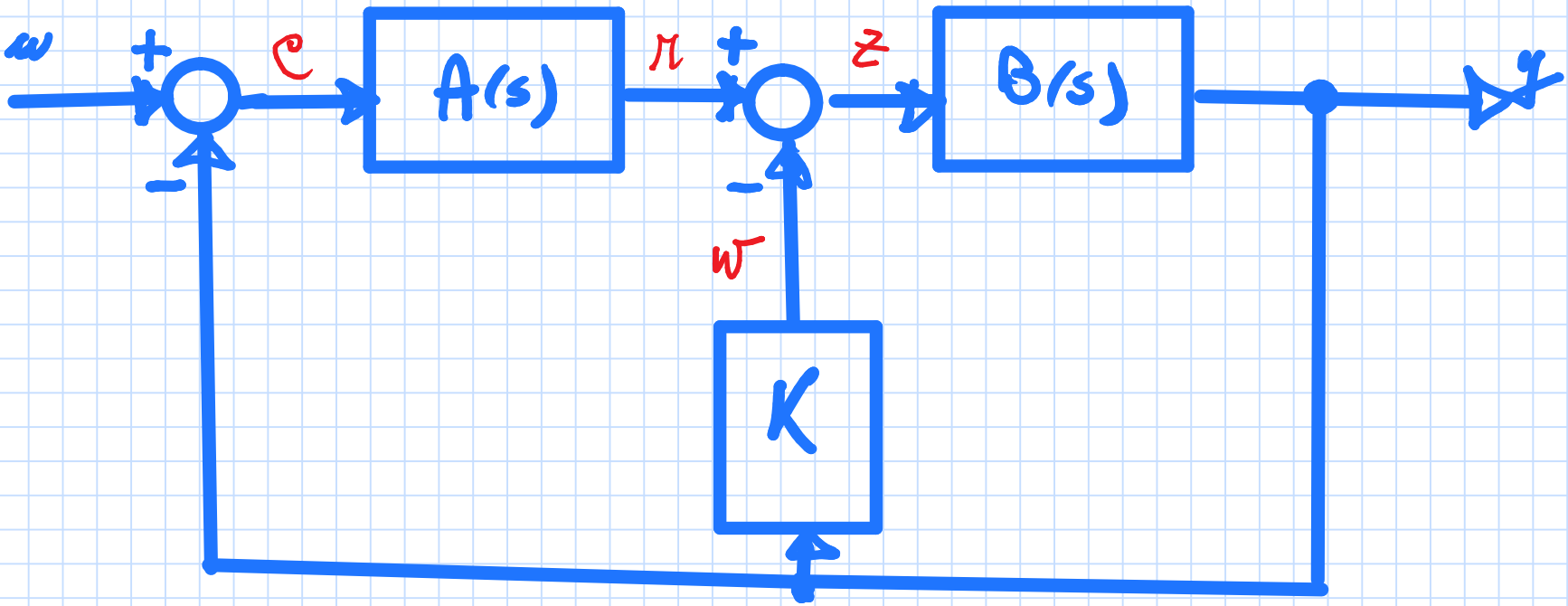


Determinare la funzione di trasferimento tra il segnale d'ingresso $u(t)$ ed il segnale d'uscita $y(t)$: $T_{u,y}(s)$



1° modo: faccio uso di segnali ausiliari per scrivere relazioni I/O per ciascun blocco e relazioni algebriche per segnali che entrano/escono dai nodi

riduzione a blocchi con le consuete ausiliarie in cascata



Ho evidenziato i segnali ausiliari. Ora determino le relazioni, usando la trasformata di Laplace

$$R(s) = A(s) \cdot E(s)$$

$$E(s) = U(s) - Y(s)$$

...

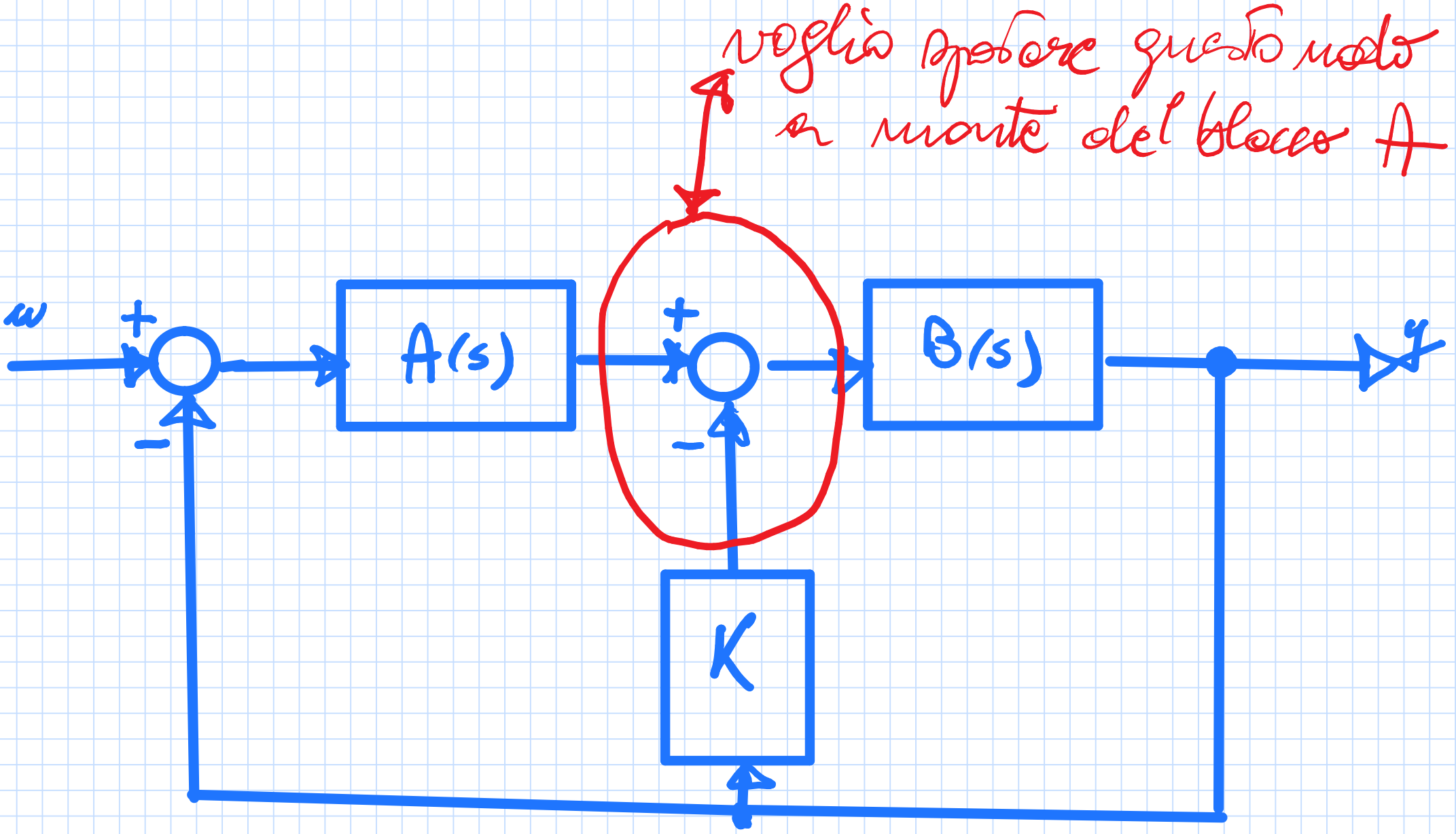
Si ottiene

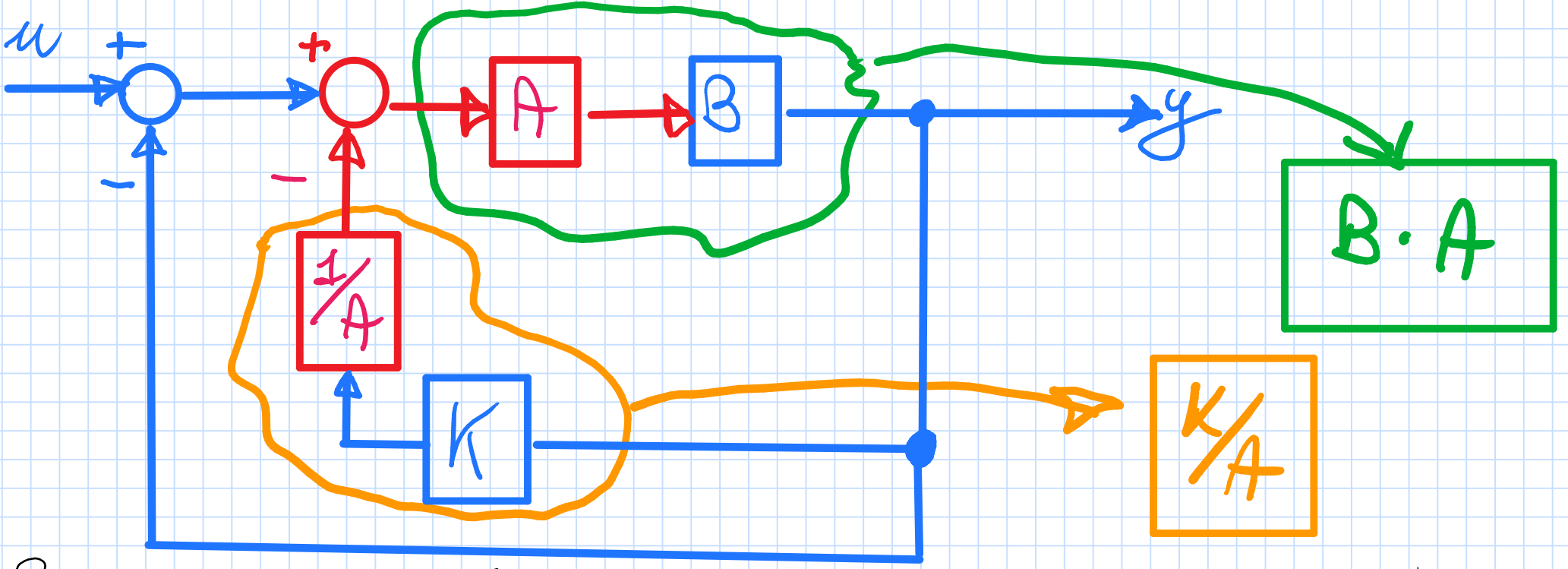
$$\begin{cases} R(s) = A(s) \cdot E(s) \\ Y(s) = B(s) \cdot Z(s) \\ W(s) = K(s) \cdot Y(s) \\ Z(s) = R(s) - W(s) \\ E(s) = U(s) - Y(s) \end{cases}$$

Ora vanno eliminate (per sostituzione nelle espressioni trovate) le trasformate di Laplace dei segnali esterni

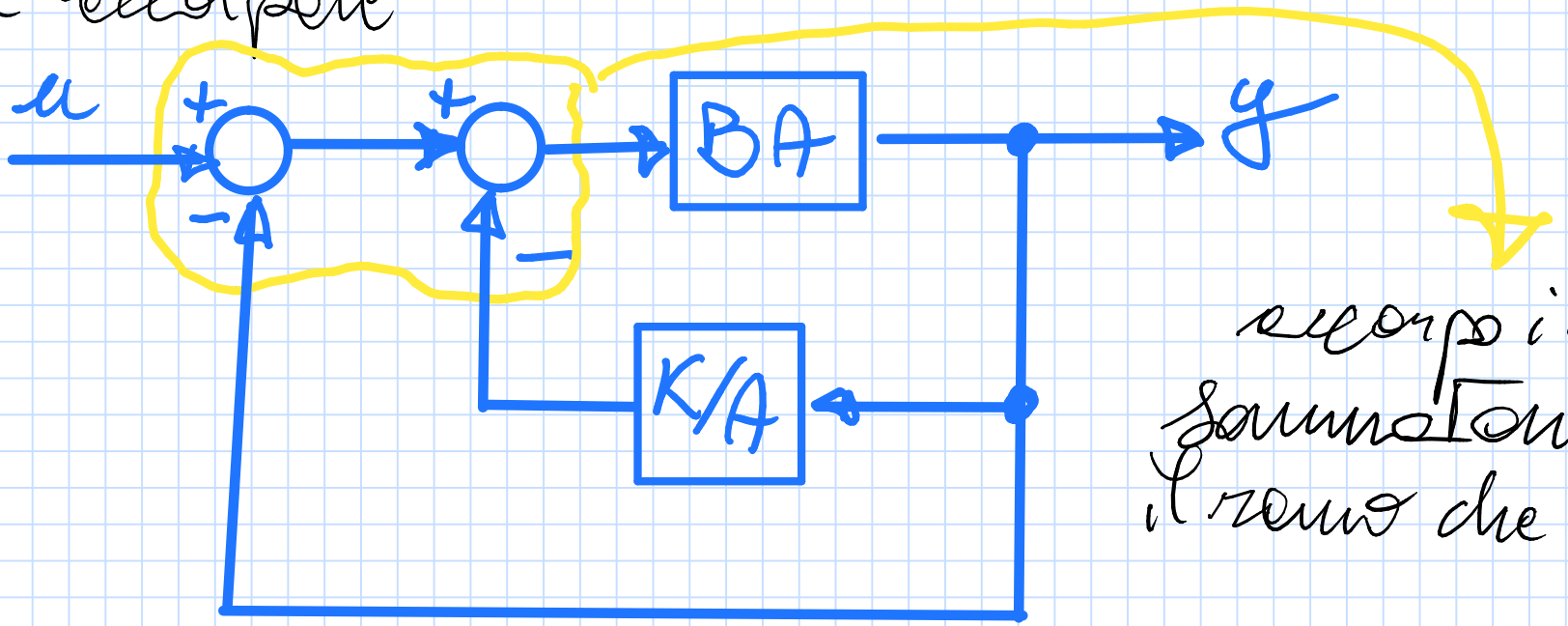
$$Y(s) = \frac{BA}{1+B[A+K]} \cdot U(s) \Rightarrow T_{uy}(s) = \frac{B \cdot A}{1+B(A+K)}$$

2° modo riscrivo graficamente lo schema a blocchi

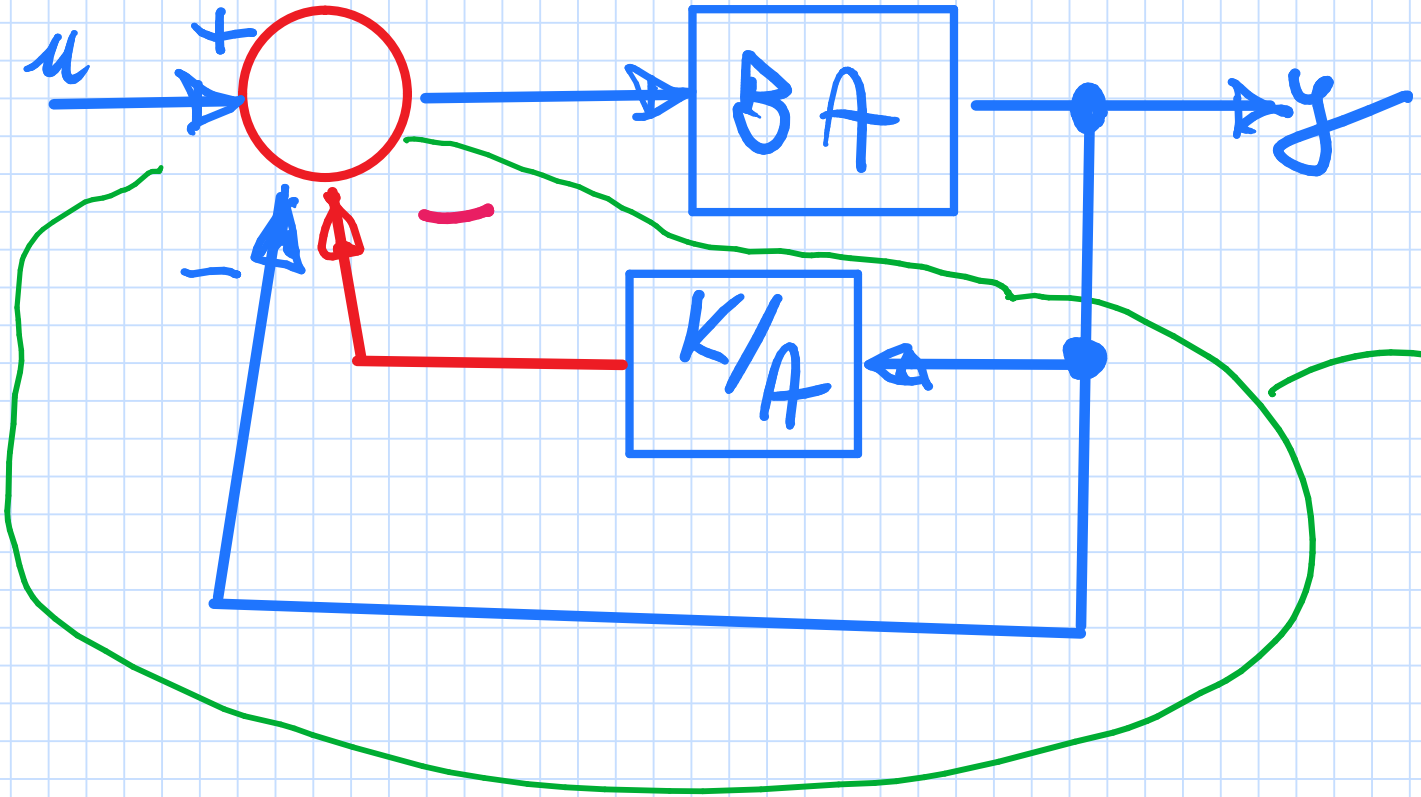




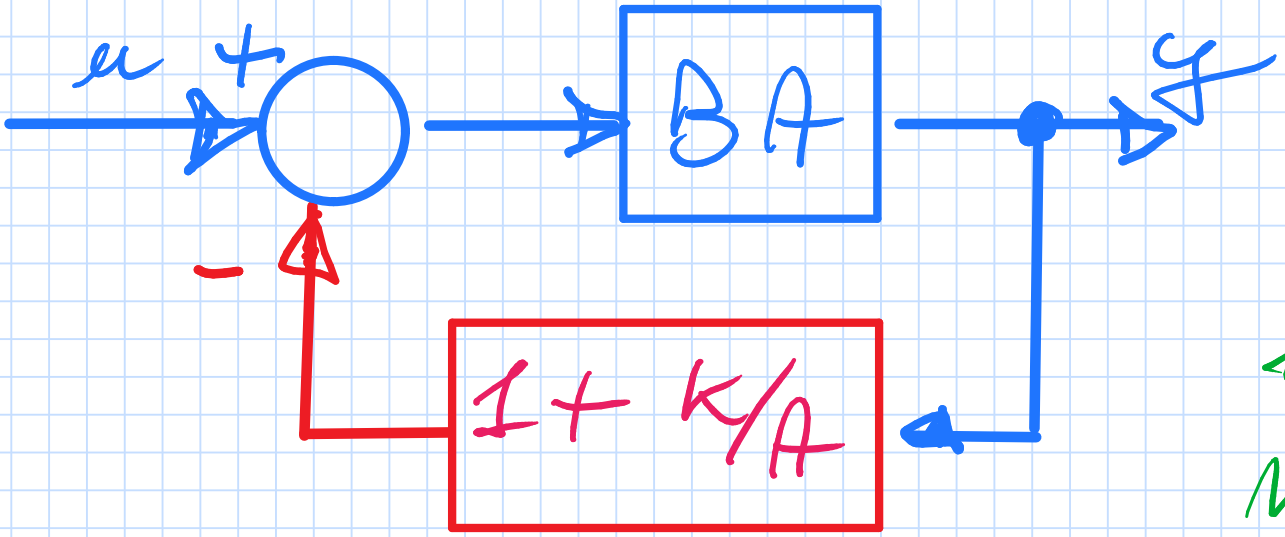
Riduciamo lo schema sostituendo le FdT in serie con quelle ricorpete



ricorpo i 2 nodi sommatori, togliendo il ramo che li collega

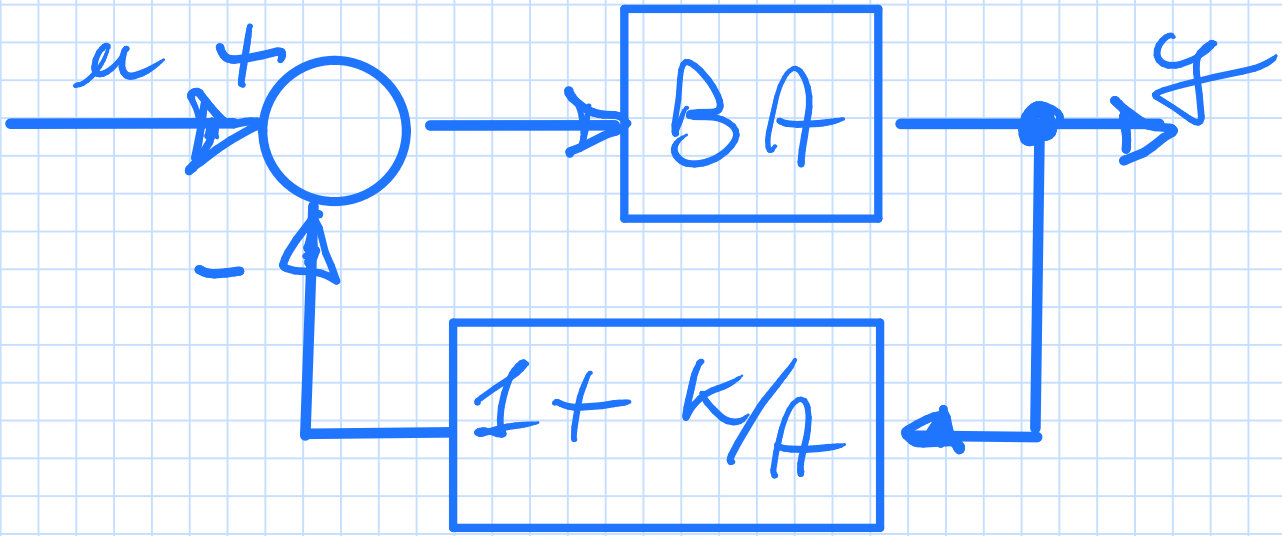


c'è una connessione "parallelo"



connessione in retroazione negativa

In definitiva la FdT cercata è

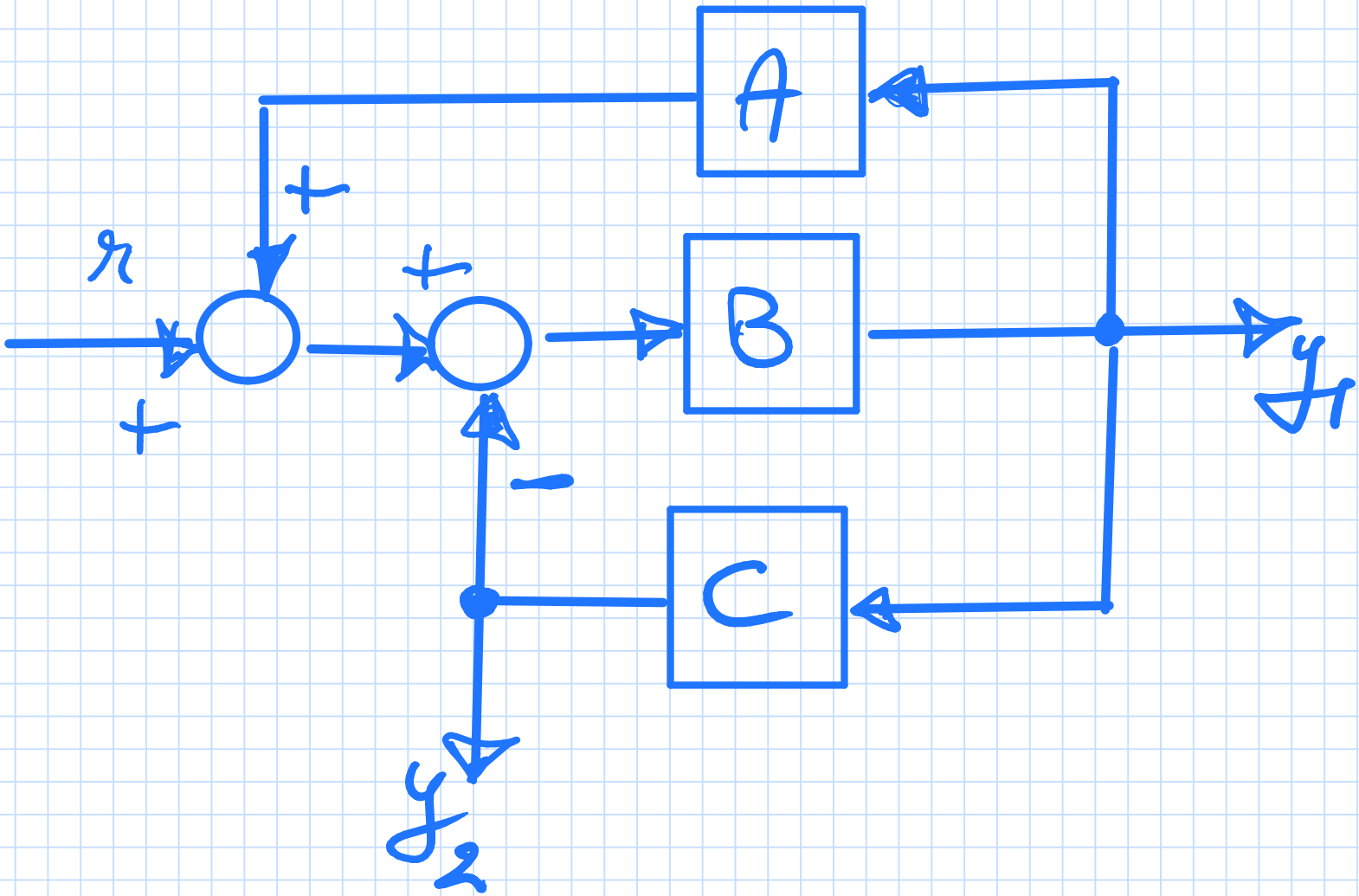


$$T_{ug} = \frac{BA}{1 + BA \cdot \frac{A+K}{A}}$$

$$= \frac{BA}{1 + B(A+K)}$$

2° esercizio

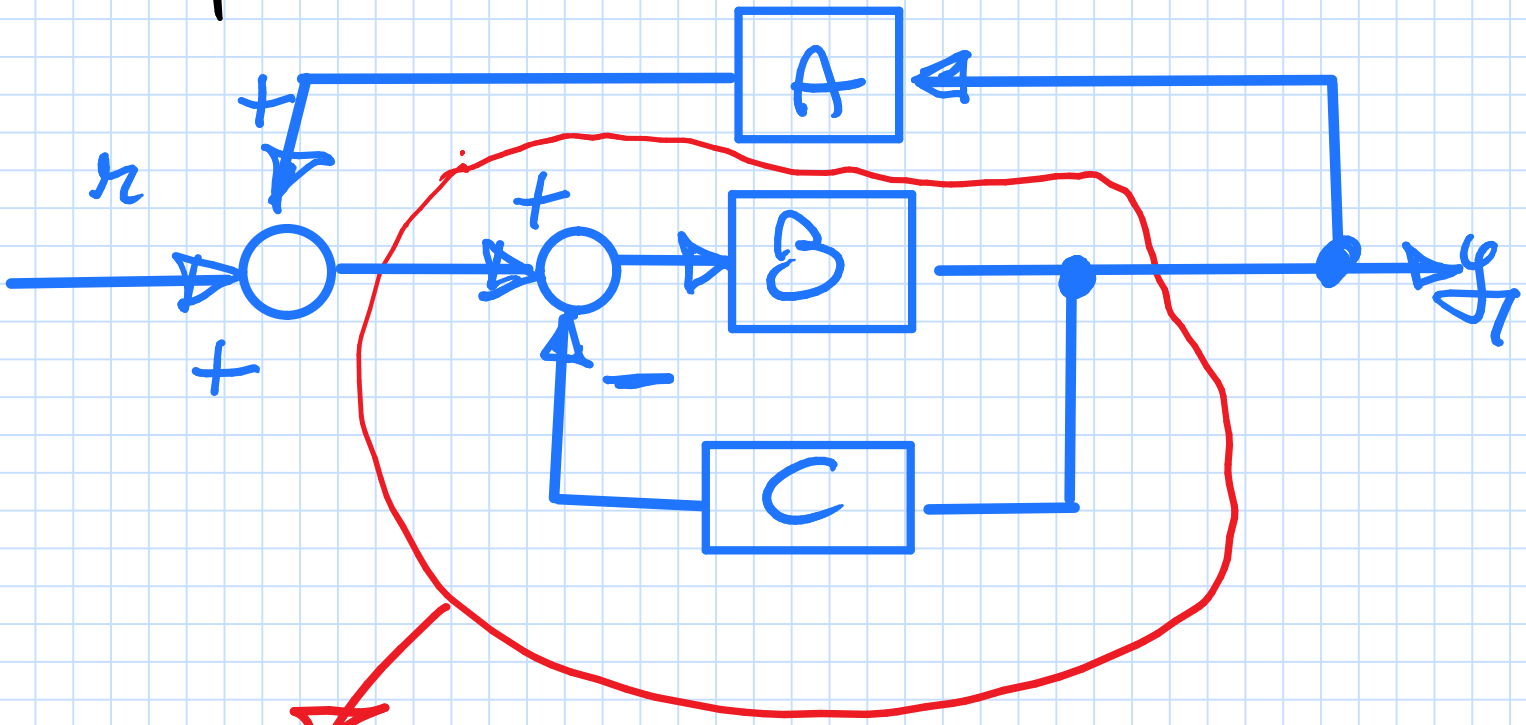
Dato lo schema a blocchi



determinare le funzioni di trasferimento $T_{rg_1}(s)$, $T_{rg_2}(s)$

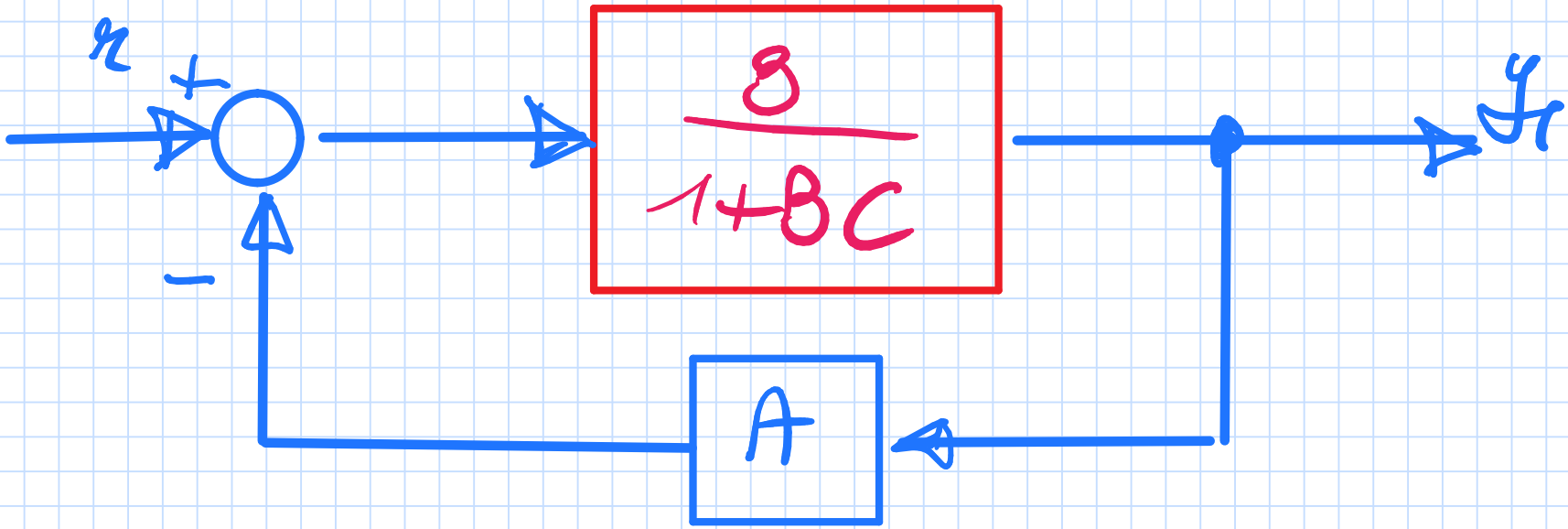
Determinazione di $T_{r,y}$

Ridisegnolo schema a blocchi considerando solo r, y



$$\frac{B}{1+BC}$$

Ridisegno lo schema dopo aver ridotto ed una FdT le serie

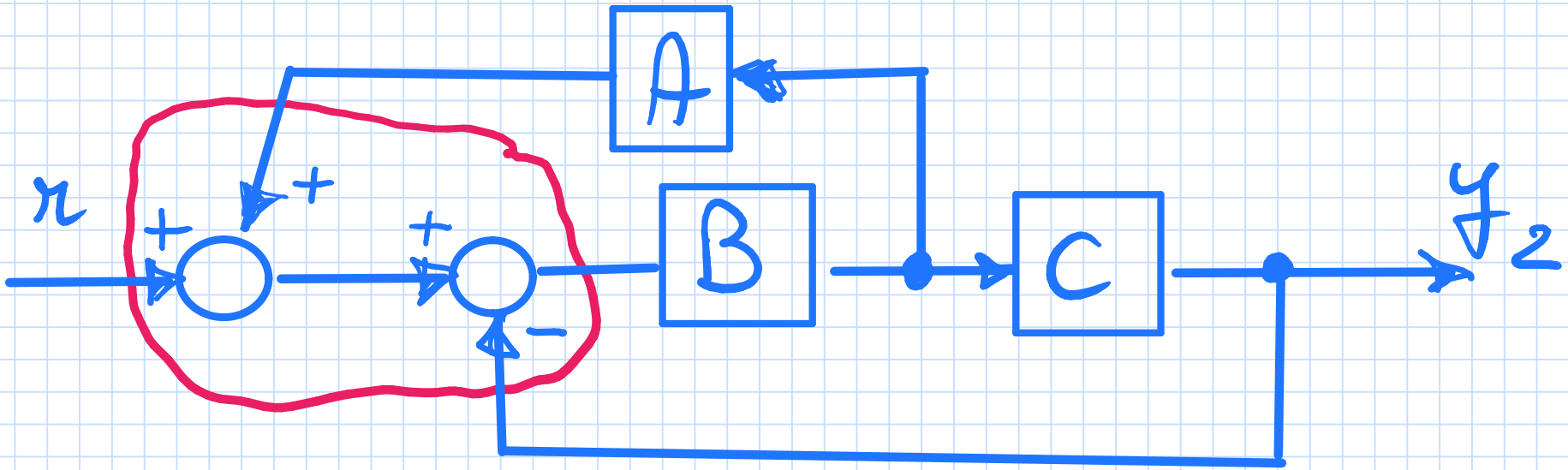


Schema z ujemnym sprzężeniem zwrotnym

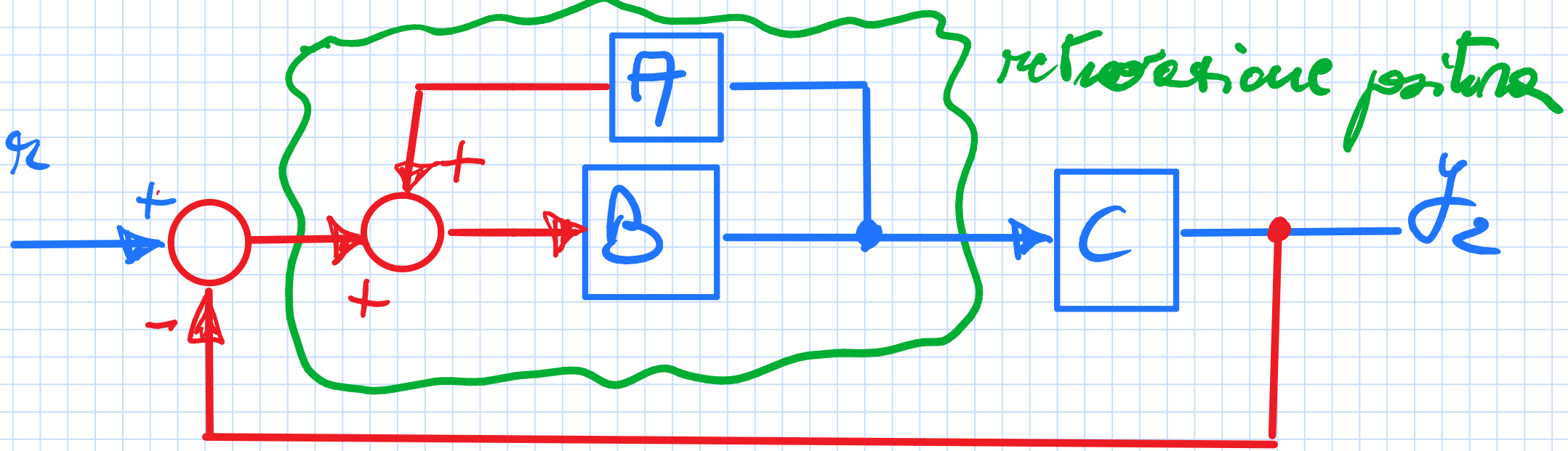
$$T_{g,y} = \frac{\frac{BA}{1+BC}}{1-\frac{BA}{1+BC}} = \frac{BA}{1+B(C-A)}$$

Determinazione di T_{r,y_2}

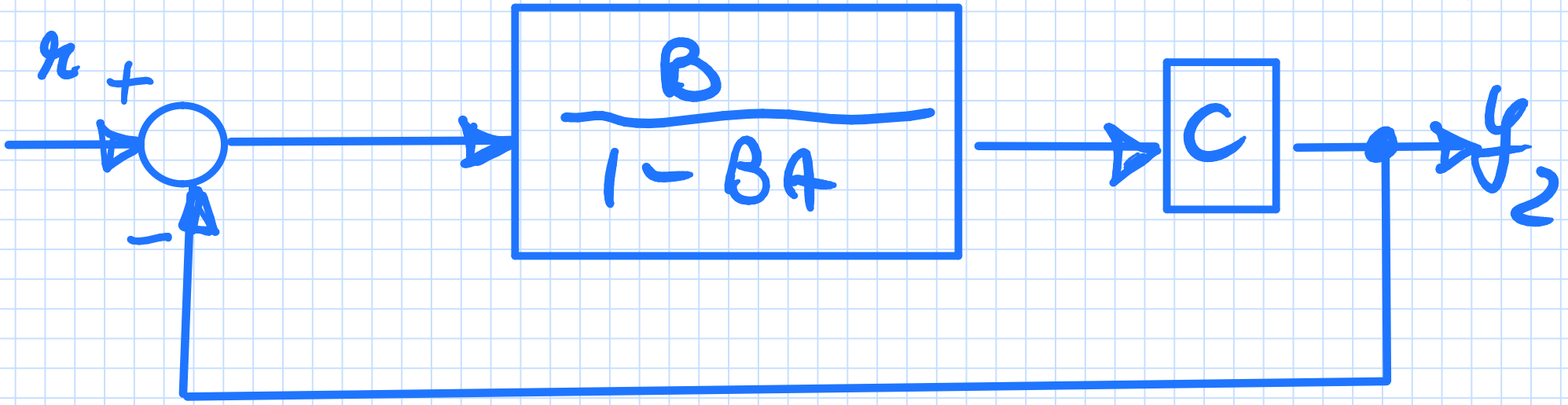
Disegno nuovamente lo schema, cambio solo r, y_2



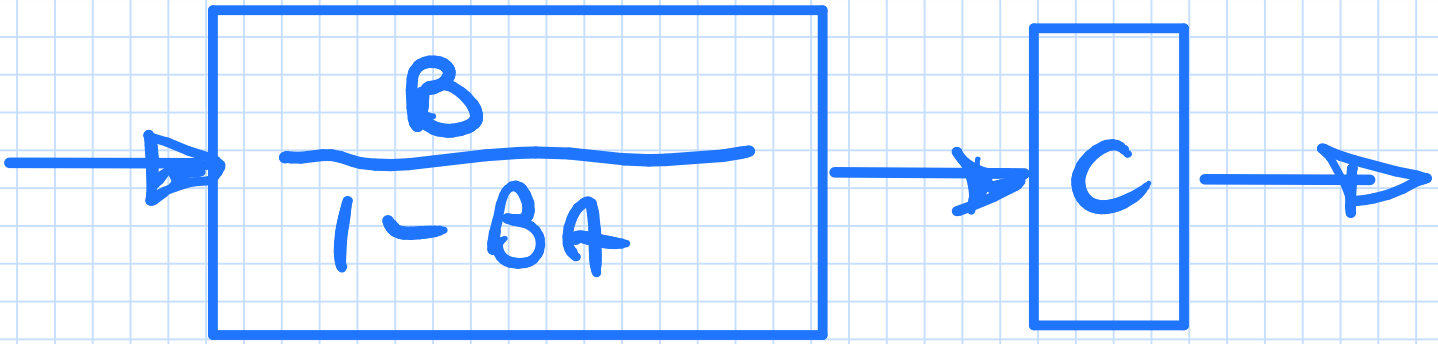
scambio di posizione
i due nodi sommatori



lo riscriviamo dopo aver semplificato la parte



Risolta la connessione - serie

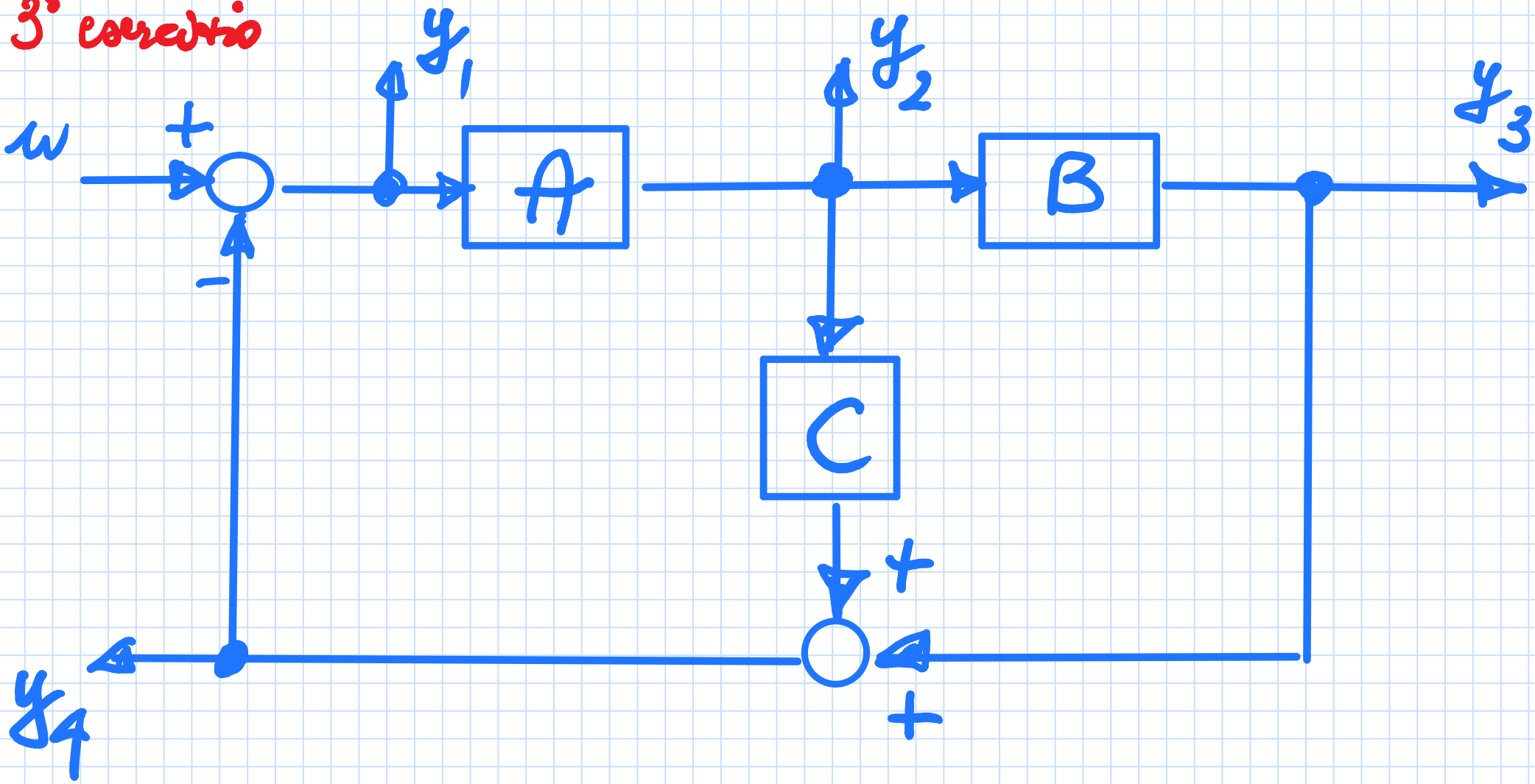


rimane una connessione di blocchi in retroazione negativa.

Per definizione:

$$T_{y_1 y_2} = \frac{BC}{1 + B(C - A)}$$

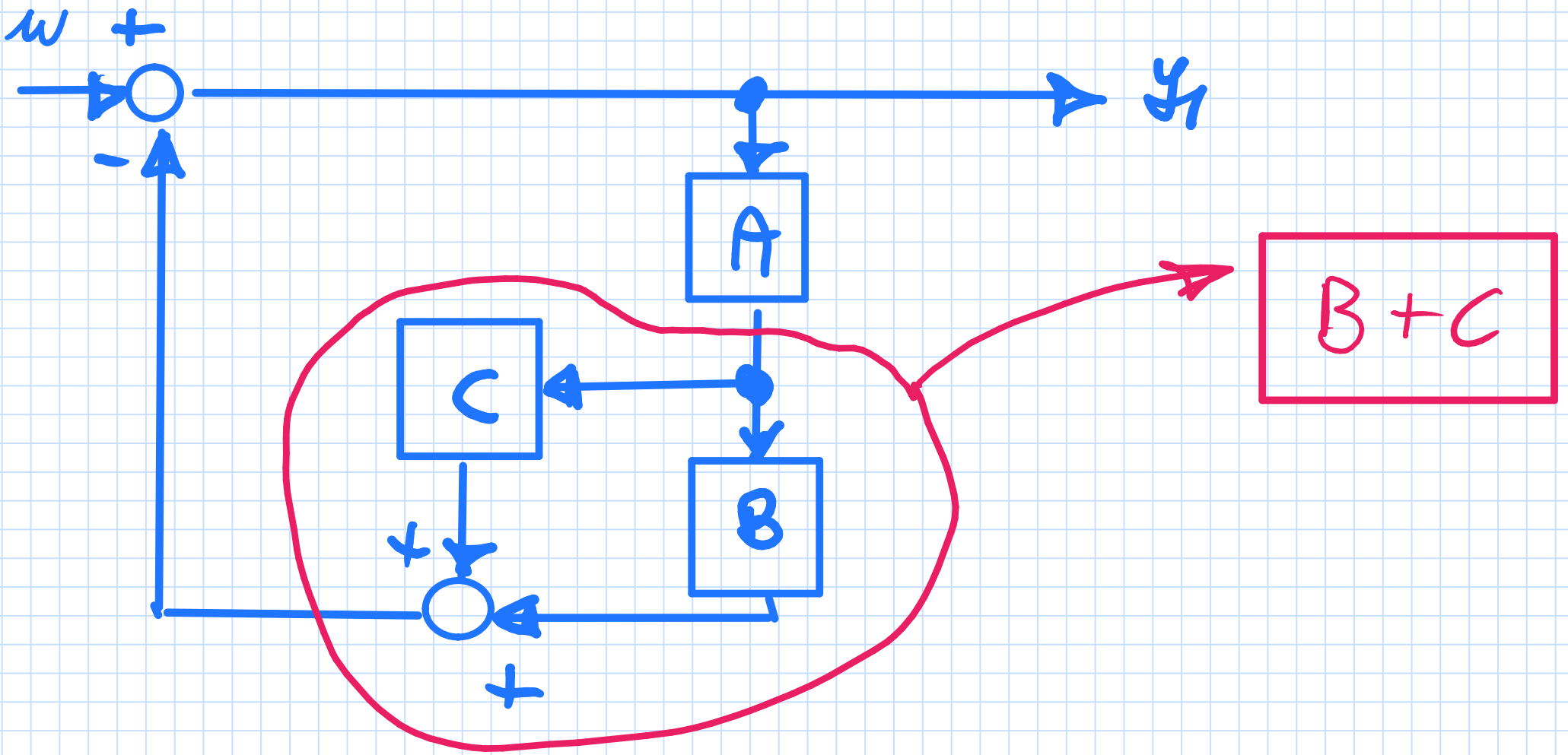
3° esercizio

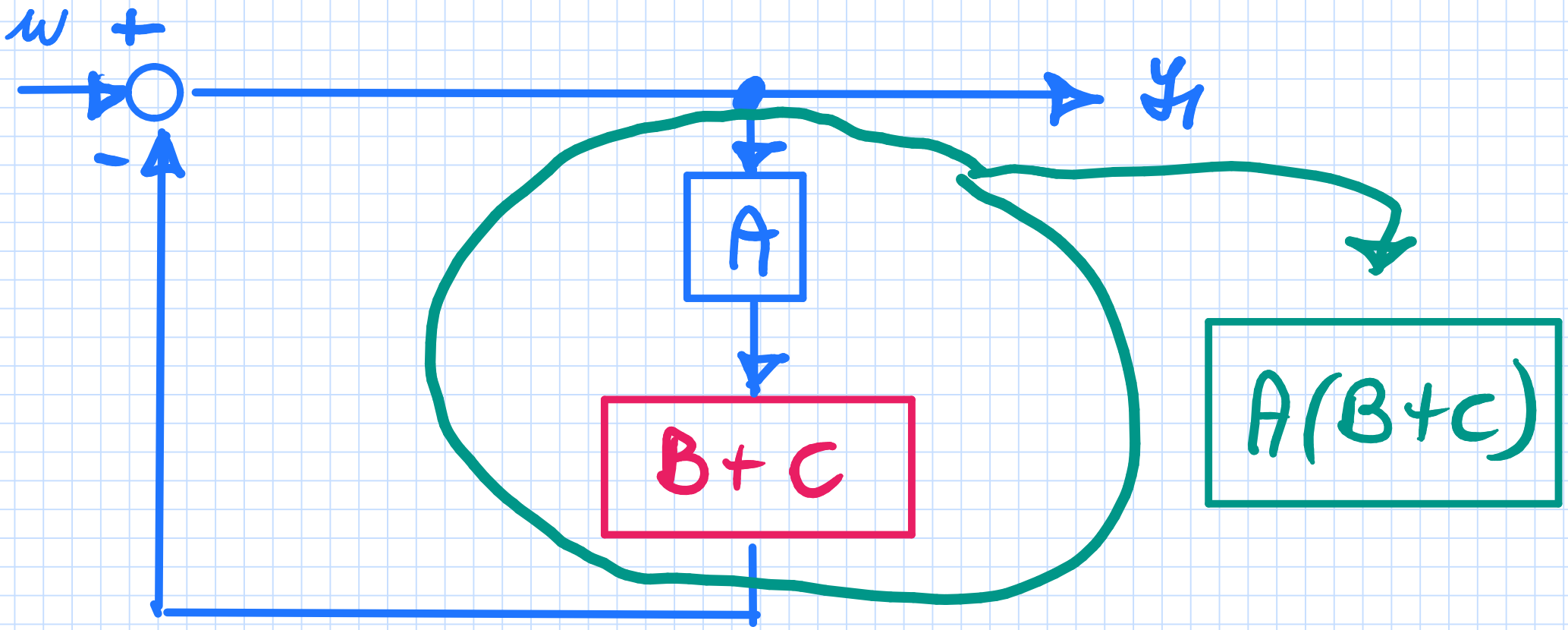


Determinare tutte le funzioni di trasferimento tra l'ingresso $w(t)$ e ciascuna delle uscite $y_1(t)$, $y_2(t)$, $y_3(t)$, $y_4(t)$

$T_1(s) = T_{u,y_1}(s)$

È conveniente riordinare lo schema a blocchi, considerando <u>solo</u> $u(t)$ ed $y_1(t)$



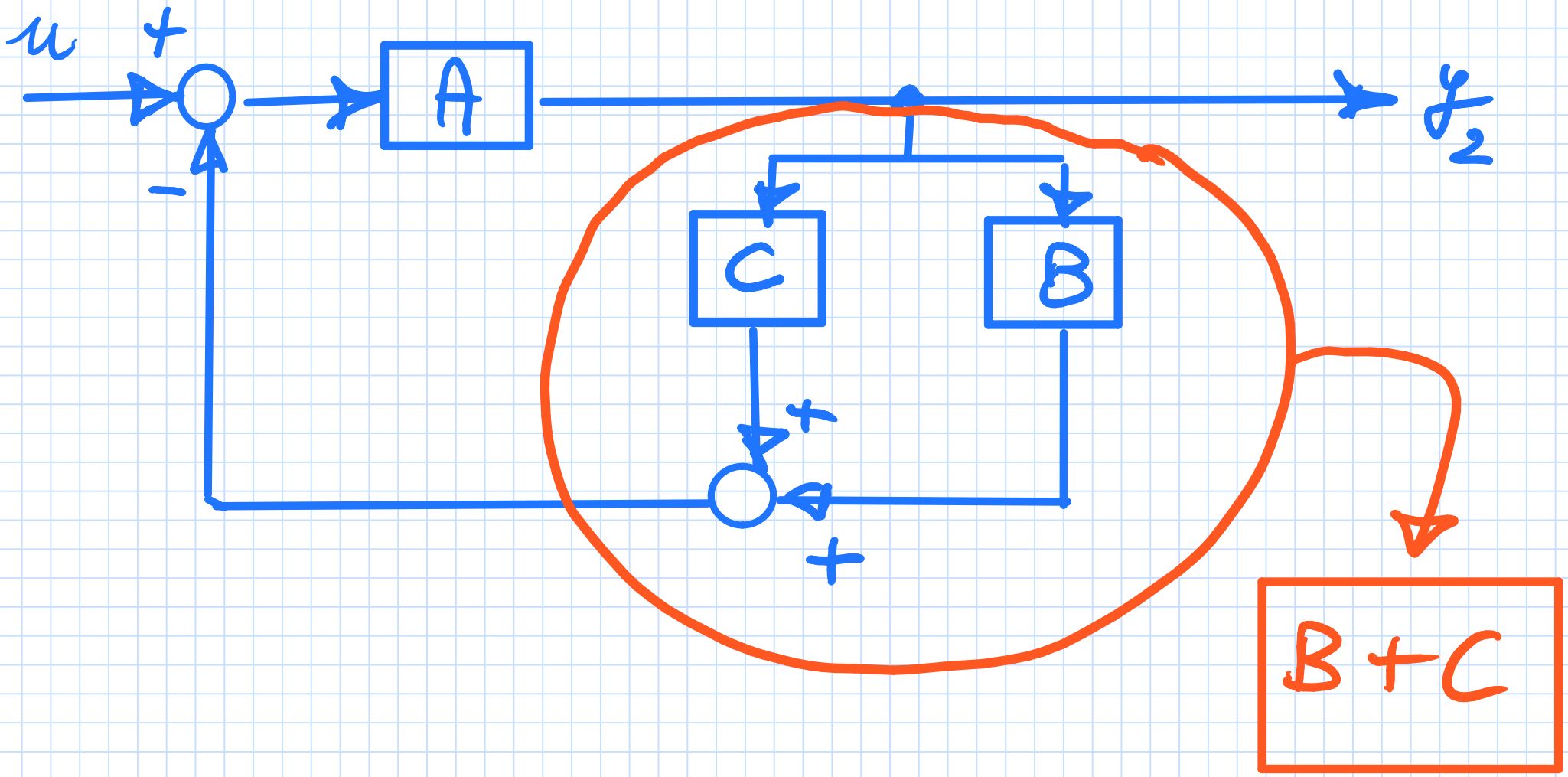


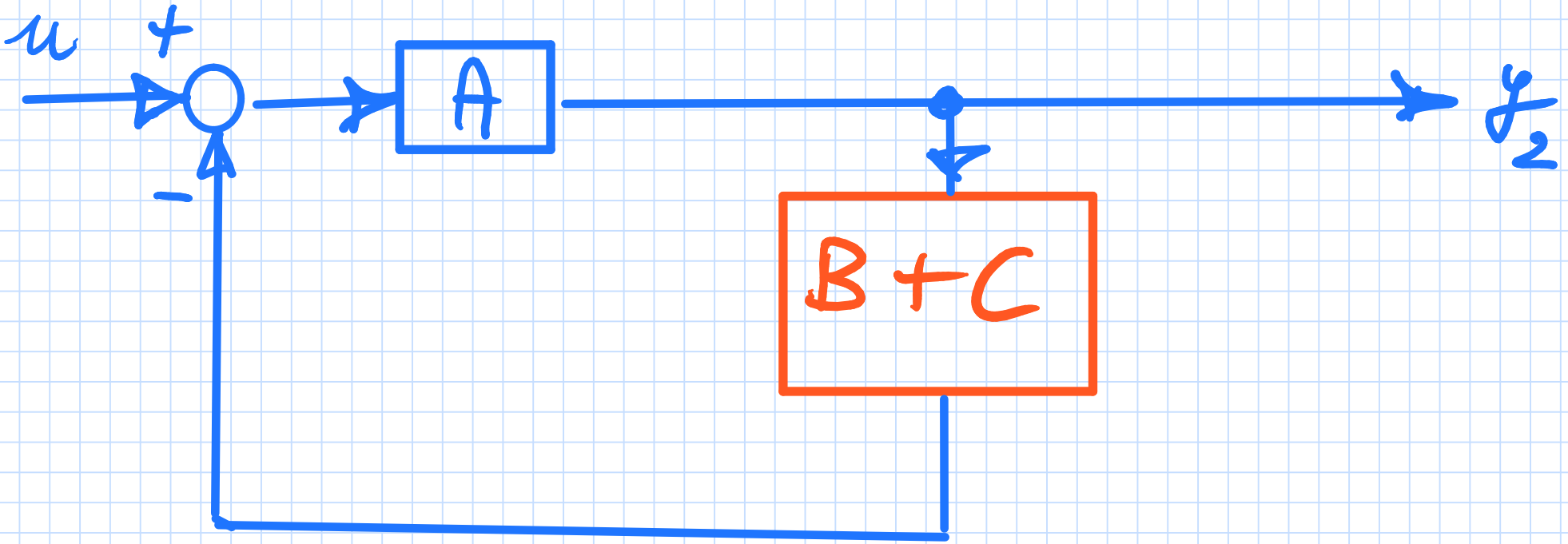
In definitiva

$$T_{u,y}(s) = \frac{1}{1 + A(B+C)}$$

$T_2(s) = T_{u,y_2}(s)$

Ridisegno lo schema a blocchi considerando <u>solo</u> u, y_2



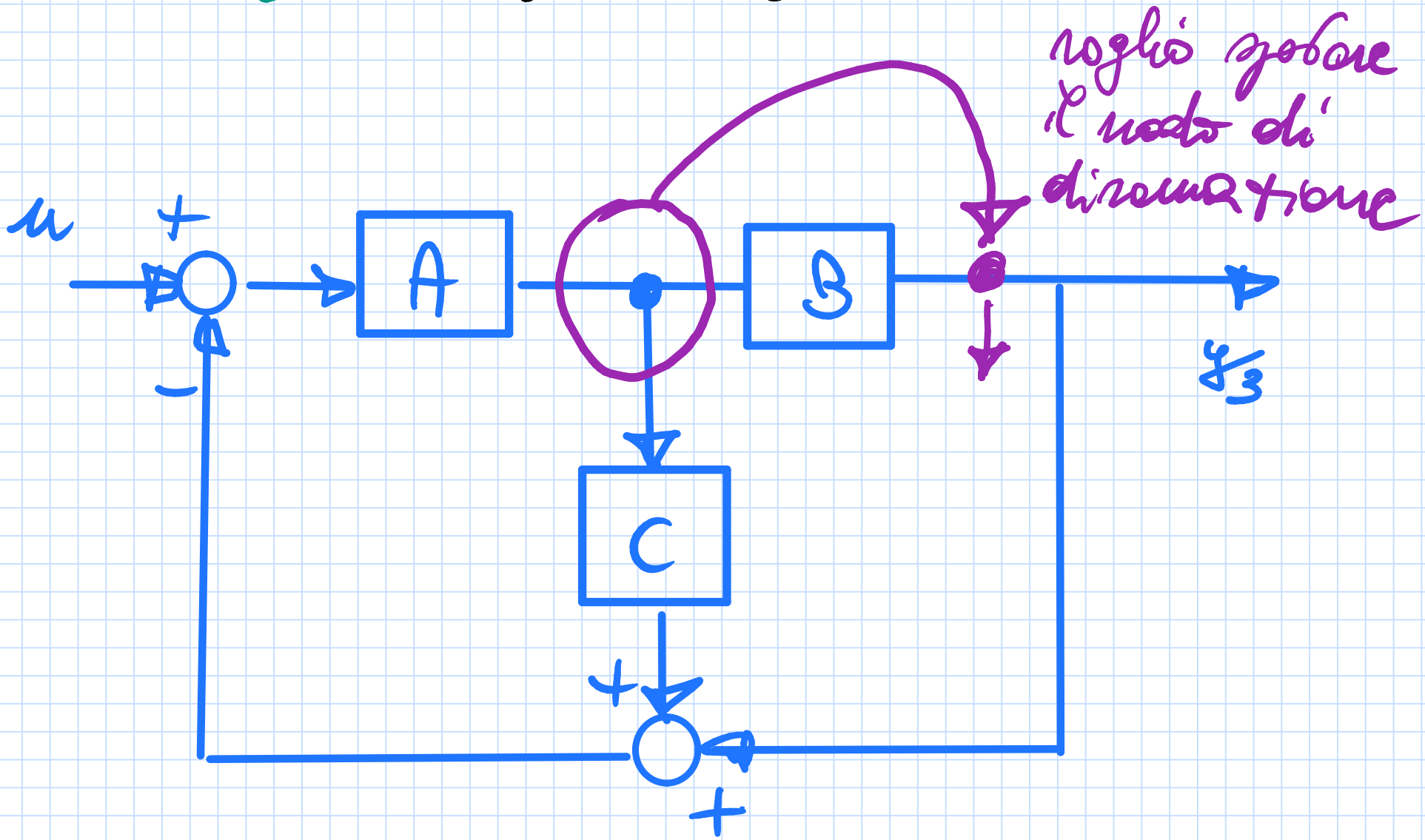


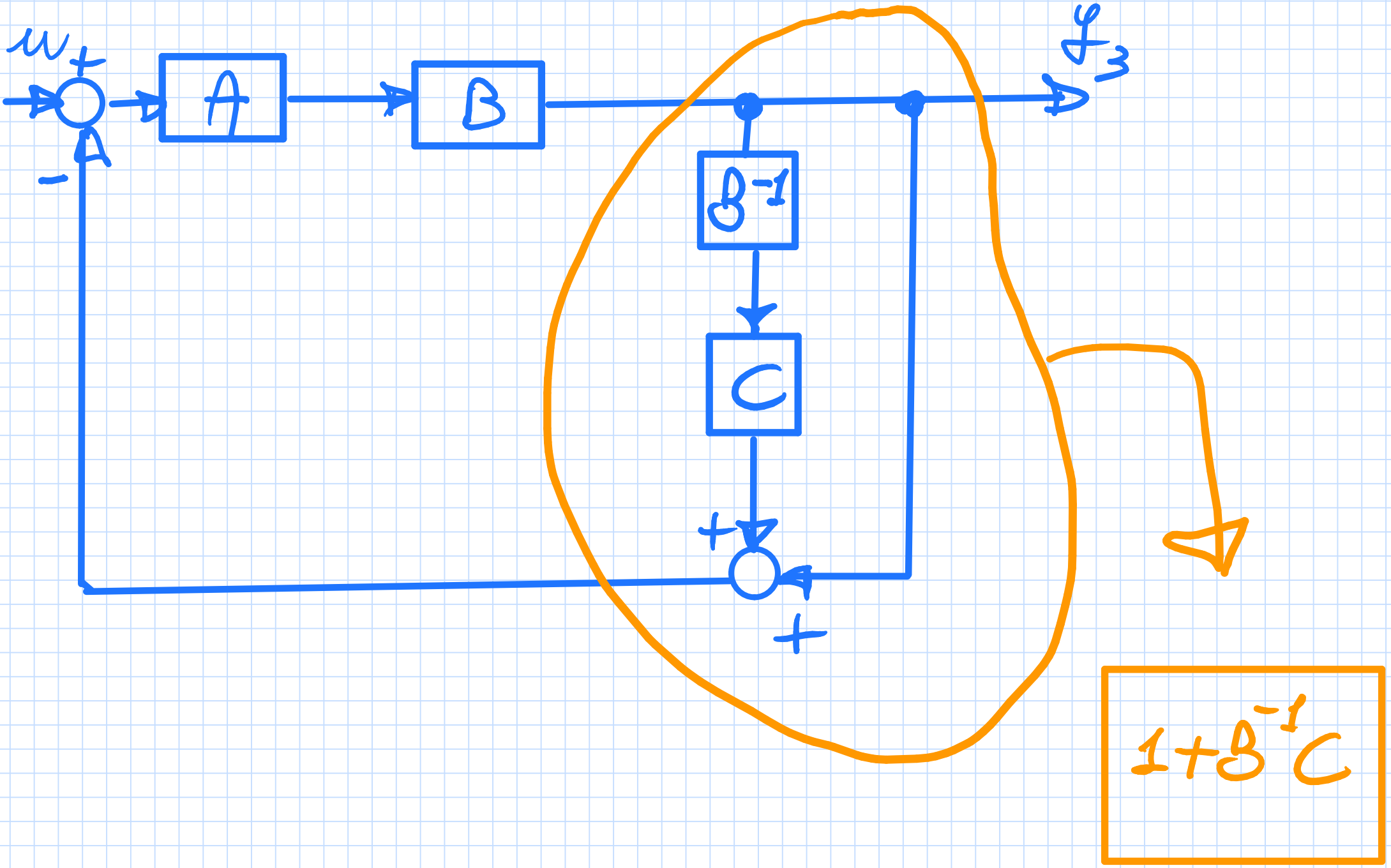
In definitiva $$T_2(s) = \frac{A}{1 + A(B+C)}$$

Vale anche che $y_2 = A y_1$ quindi è possibile determinare T_2 da T_1 molto semplicemente ($T_2 = A T_1$)

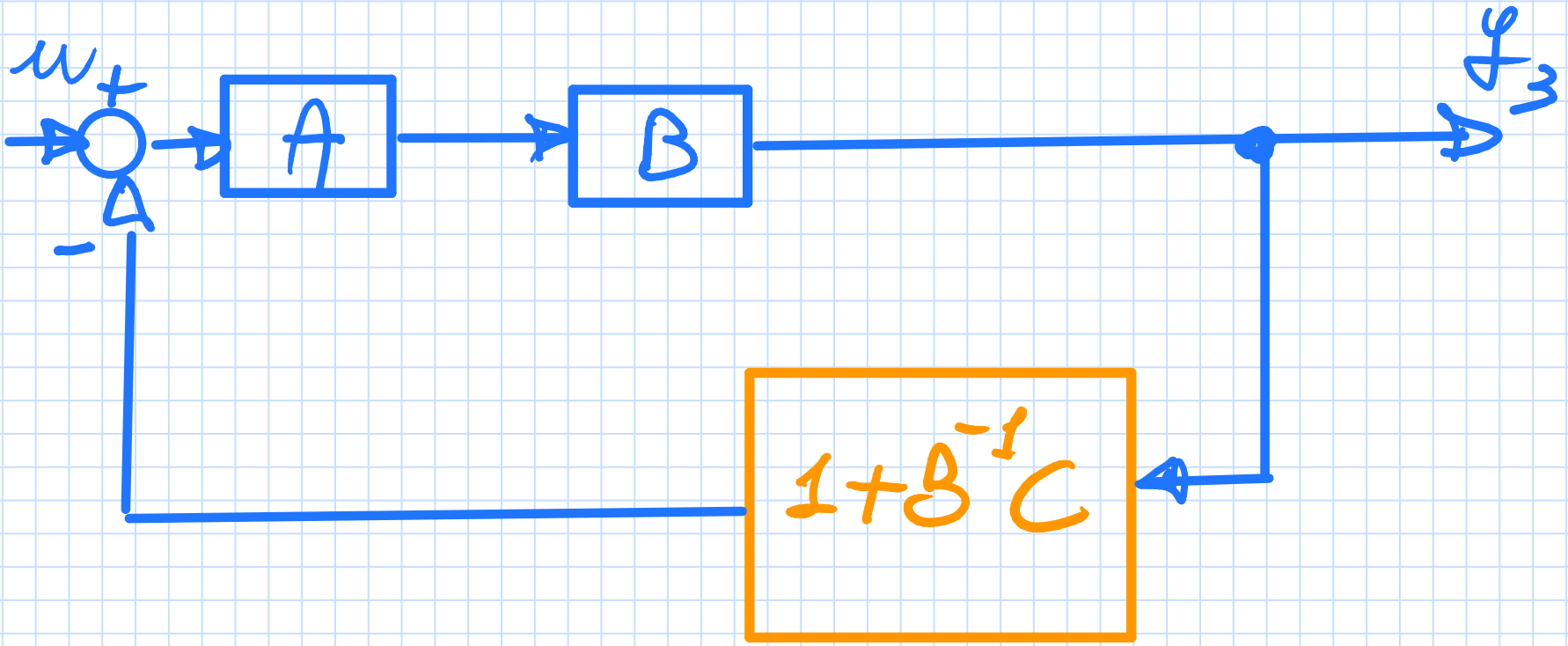
$T_3(s) = T_{u,y_3}(s)$

Ridisegnato lo schema a blocchi di ottiene





$$1+B^{-1}C$$

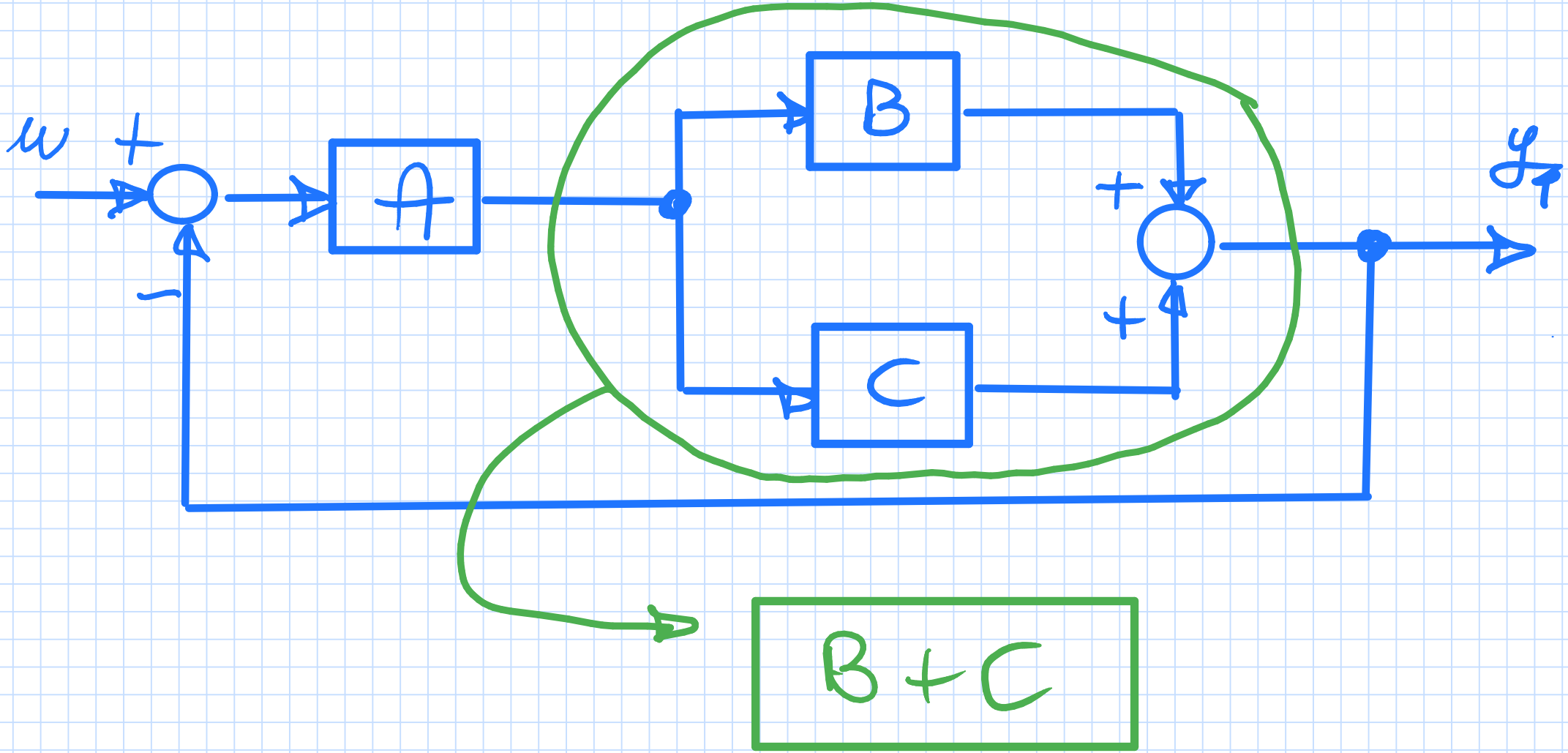


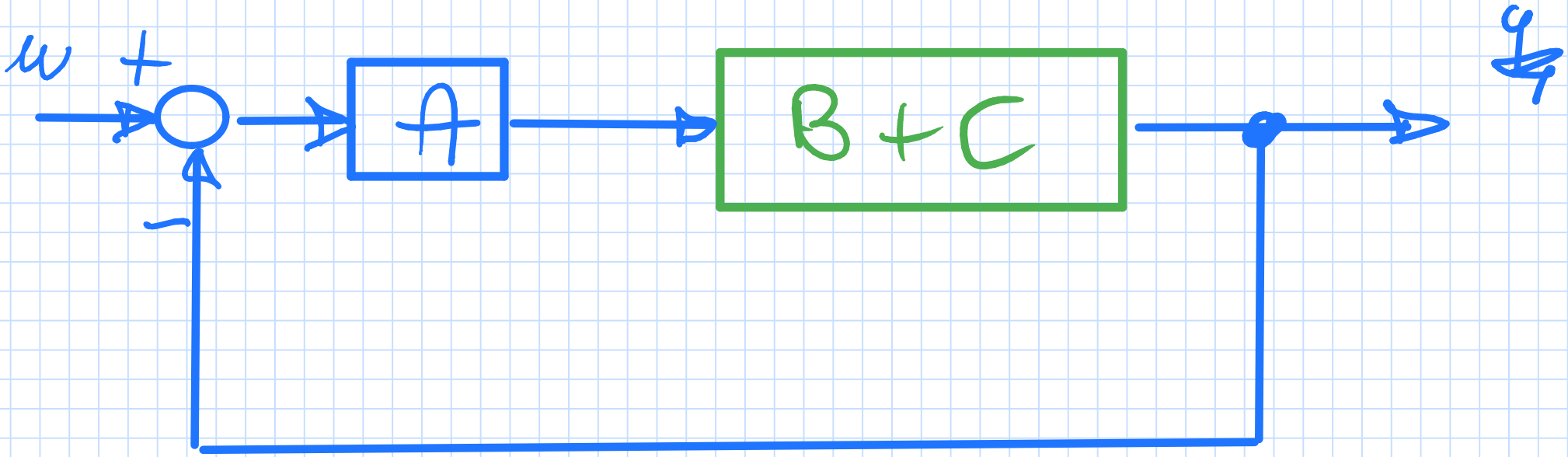
Quindi $$T_3(s) = \frac{AB}{1+AB(1+B^{-1}C)} = \frac{AB}{1+A(B+C)}$$

NB vale anche che $y_3 = B\,y_2$ [quindi $T_3 = B\,T_2$]

$T_4(s) = T_{u,y_4}(s)$

Al solito, ridisegno lo schema a blocchi con solo u, y_3





Per T_4 vale allora che $$T_4 = \frac{A(B+C)}{1+A(B+C)}$$

Vale anche che $y_4 = y_3 + C y_2$ da cui lo schema rivisitato