# Realizzazione – LTI a tempo continuo

$$\begin{cases} \dot{x} = ax + bu \\ y = cx + du \end{cases}$$

$a, b, c, d \in \mathbb{R}$

$x \to$ dimensione 1

$\Downarrow$ Laplace trasf.

$$X(s) = \frac{b}{s-a} U(s)$$

$$Y(s) = \frac{cb}{s-a} U(s) + d\,U(s) \Rightarrow T(s) = \frac{cb}{s-a} + d$$

$$= \frac{ds + (cb - da)}{s-a}$$

## Realizzazione

Da FdT di ordine 1 ad equazioni di stato

1° caso FdT strettamente propria $G(s) = \frac{\beta}{ps - q}$

$$G(s) = \frac{f}{ps - q} = \frac{f/p}{s - q/p}$$

$$\overset{?}{\Leftrightarrow}$$

$$T(s) = \frac{cb}{s - a} \quad \text{(d = 0)}$$

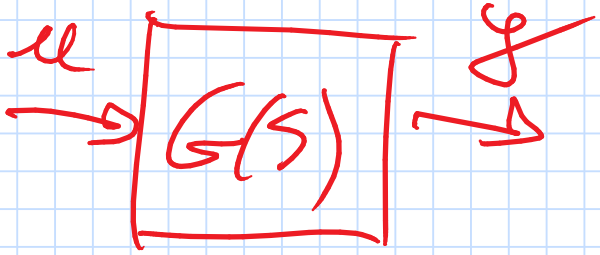
$$\begin{cases} f/p = c \cdot b \\ q/p = a \end{cases} \Leftarrow \infty \text{ volte!}$$

$c = 1 \quad b = f/p$

$b = 1 \quad c = f/p$

$$G(s) = \frac{2}{3s+5} = \frac{2/3}{s+5/3} \qquad \frac{c \cdot b}{s-a}$$

$$\begin{cases} c \cdot b = 2/3 \\ a = -5/3 \end{cases} \qquad c = 1 \quad b = 2/3 \qquad a = -5/3$$




$$\begin{cases} \dot{x} = -\frac{5}{3}x + \frac{2}{3}u \\ y = x \end{cases}$$

FdT ricavata propria

$$G(s) = \frac{ps - q}{rs - w}$$

$$T(s) = \frac{cb}{s-a} + d$$

$$\begin{array}{l|l} ps - q & rs - w \\ \hline ps + \frac{wp}{r} & p/r \\ \hline \frac{wp - qr}{r} \triangleq M & \end{array}$$

$$\frac{p}{r} + \frac{M}{rs - w}$$

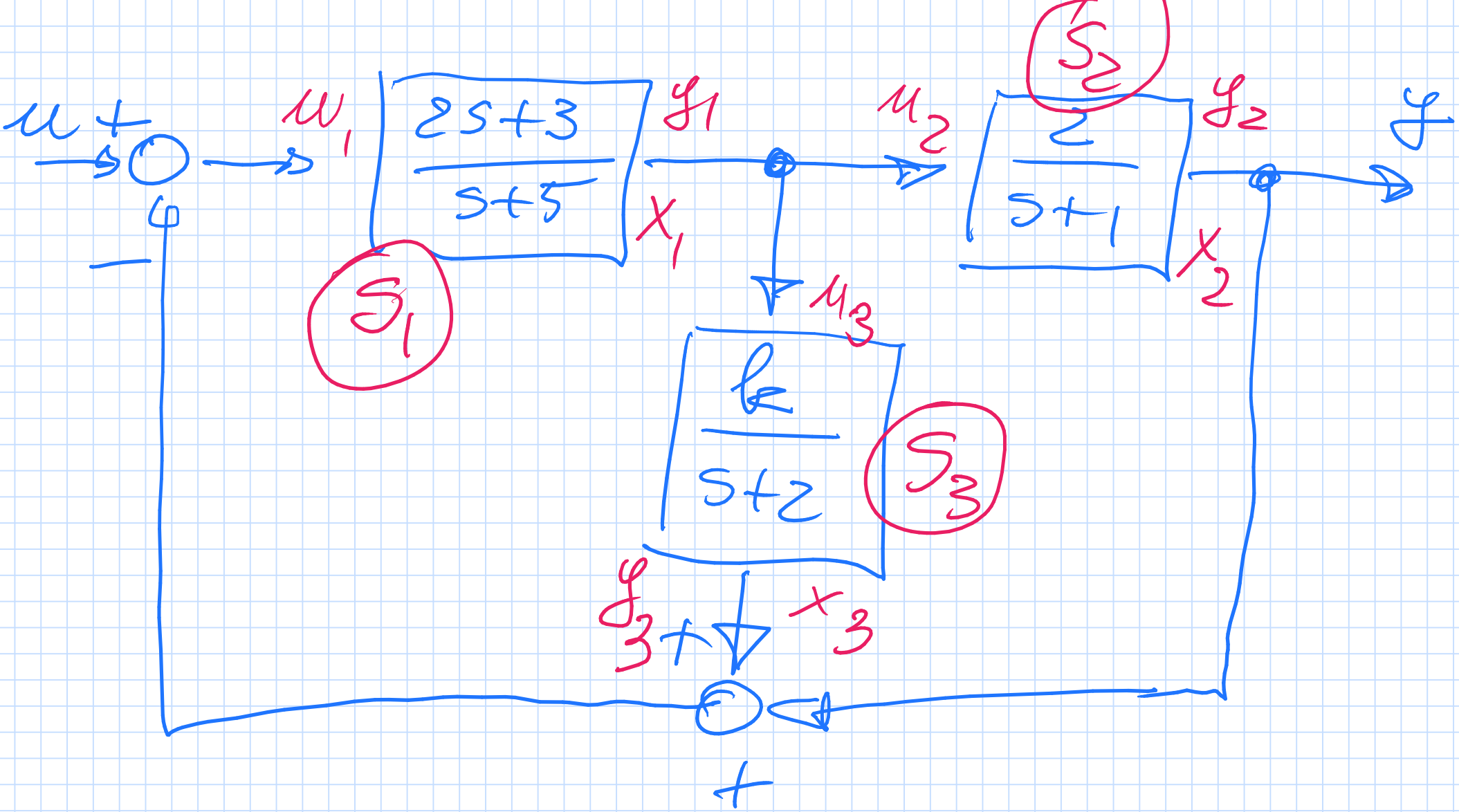
$$G(s) = \frac{2s+3}{s+5} = -\frac{7}{s+5} + 2$$

$$T(s) = d + \frac{cb}{s-a}$$

$$\begin{array}{r|l} 2s+3 & s+5 \\ \hline -2s-10 & 2 \\ \hline -7 & \end{array}$$

$$\begin{cases} \dot{x} = -5x - 7u \\ y = x + 2u \end{cases}$$

$$\begin{cases} d = 2 \\ a = -5 \\ cb = -7 \end{cases} \qquad \begin{array}{l} c = 1 \\ b = -7 \end{array}$$


u
+
−
+
u1
2s+3
s+5
y1
x1
S1
u2
2
s+1
S2
y2
y
x2
u3
k
s+2
S3
y3
x3
+
+

$S_1$

$$\frac{2s+3}{s+5}$$

$$\begin{cases} \dot{x}_1 = -5x_1 - 7u_1 \\ y_1 = x_1 + 2u_1 \end{cases}$$

$S_2$

$$\frac{2}{s+1}$$

$$\begin{cases} \dot{x}_2 = -x_2 + 2u_2 \\ y_2 = x_2 \end{cases}$$

$S_3$

$$\frac{k}{s+2}$$

$$\begin{cases} \dot{x}_3 = -2x_3 + k u_3 \\ y_3 = x_3 \end{cases}$$

$$\dot{x}_1 = -5x_1 - 7u_1$$

$$\dot{x}_2 = -x_2 + 2u_2$$

$$\dot{x}_3 = -2x_3 + 6u_3$$

$$u_1 = u - x_2 - x_3 = u - y_2 - y_3$$

$$u_2 = y_1 = x_1 + 2u_1 = x_1 - 2x_2 - 2x_3 + 2u$$

$$u_3 = y_1 = x_1 + 2u_1 = x_1 - 2x_2 - 2x_3 + 2u$$

$$y_1 = x_1 + 2u_1$$

$$y_2 = x_2$$

$$y_3 = x_3 \qquad y = y_2 = x_2$$

$$\begin{cases} \dot{x}_1 = -5x_1 + 7x_2 + 7x_3 - 7u \\ \dot{x}_2 = +2x_1 - 5x_2 - 4x_3 + 4u \\ \dot{x}_3 = kx_1 - 2kx_2 - 2(k+1)x_3 + 2ku \\ y = x_2 \end{cases}$$

$$G(z) = \frac{40}{27z + 13}$$

$$= \frac{40/27}{z + 13/27}$$

$$T(z) = \frac{cb}{z - a}$$

$$\begin{cases} c \cdot b = 40/27 \\ a = -13/27 \end{cases}$$

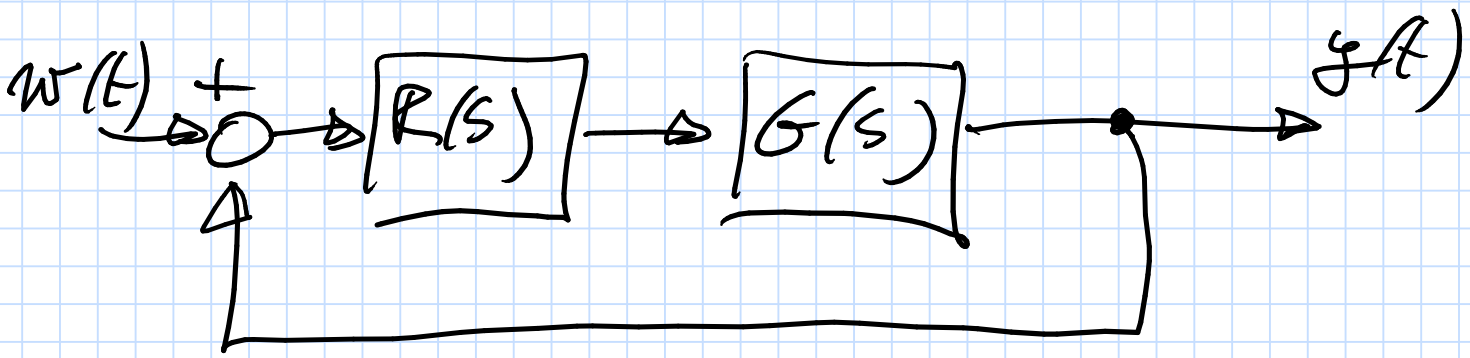
$c = 1 \quad b = \dots$

$b = 1 \quad c = \dots$

$$\begin{cases} x(k+1) = a\,x(k) + b\,u(k) \\ y(k) = c\,x(k) + d\,u(k) \end{cases} \qquad x(0) = 0$$

$$T(z) = \frac{cb}{z-a} + d$$

9/6/2014




$$G(s) = \frac{1{,}5\,(1+s)}{s\,(1+20s)}$$

R(s) fisr. realizzabile

Vale che

a) $e_\infty \to 0$ per $w(t) = 1(t)$

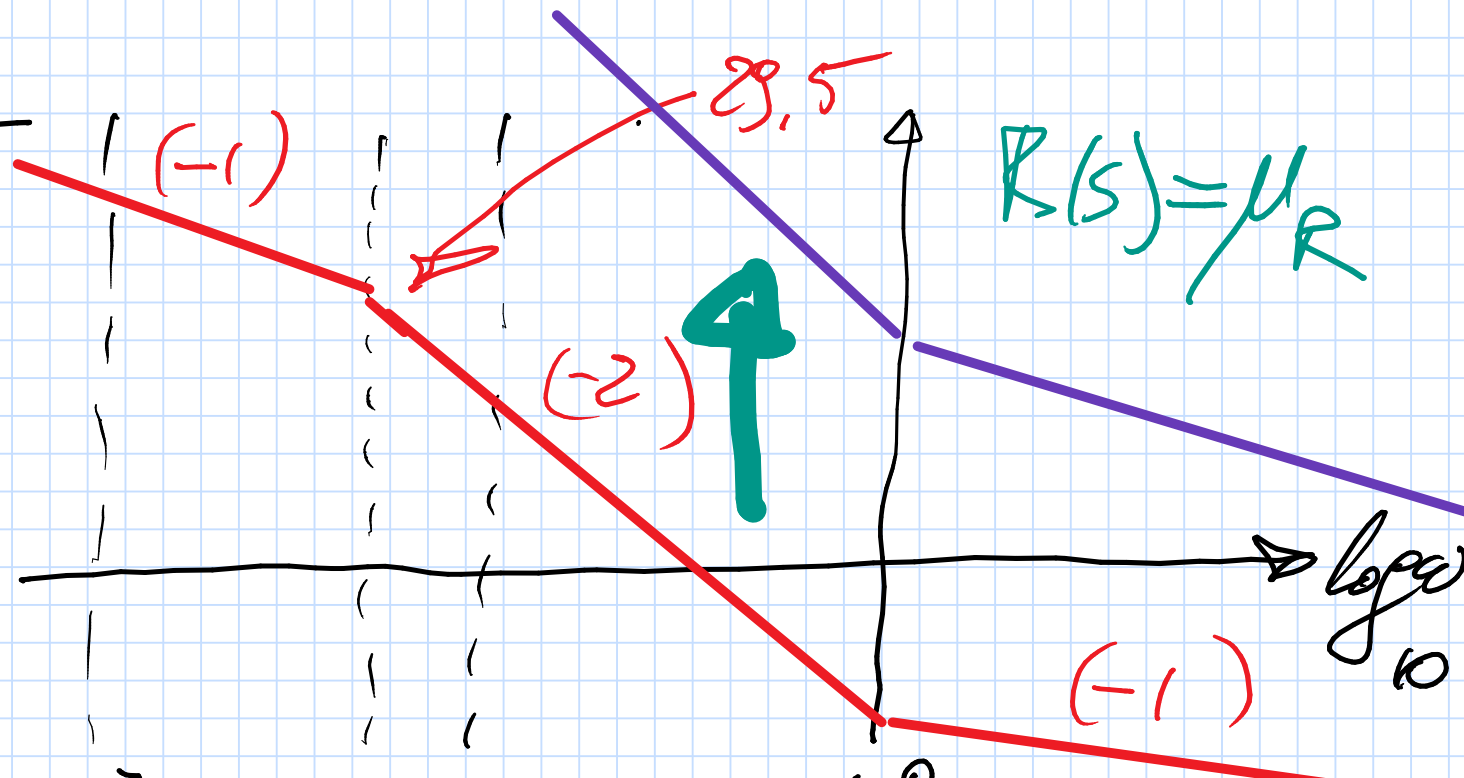
b) $\omega_c \geq 5$ rad/s

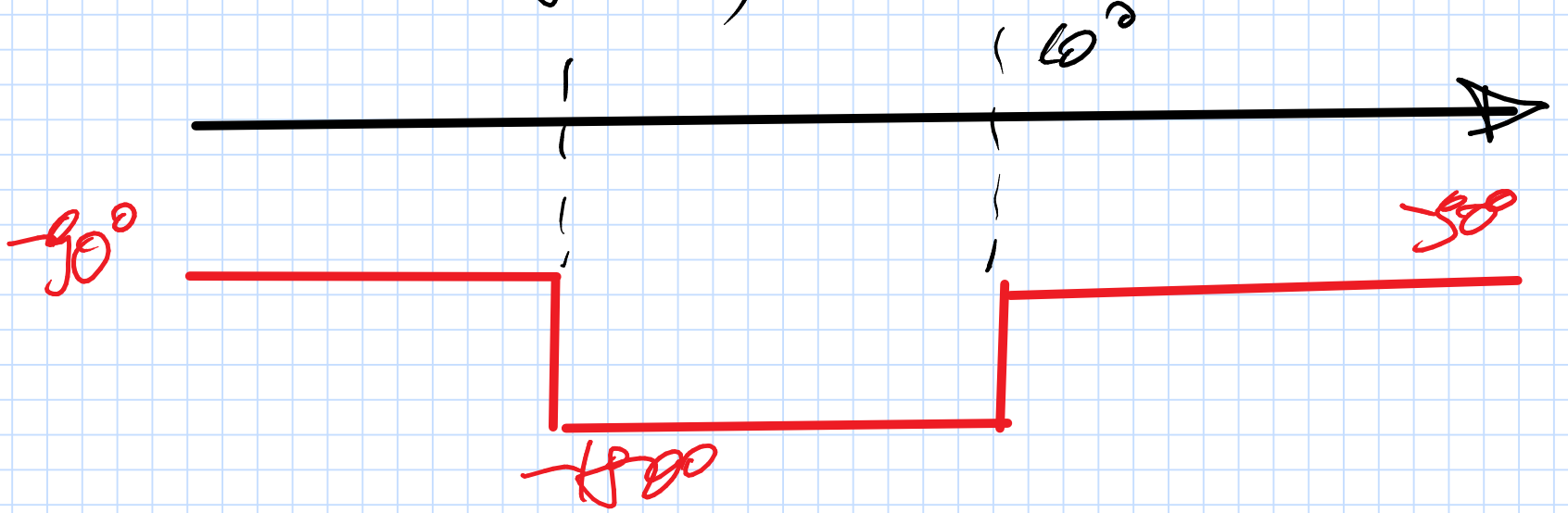
c) $\varphi_m \geq 45°$

$$G(s) = \frac{1{,}5(1+s)}{s(1+20s)}$$

$p = 0 \qquad z = -1$

$p = -\frac{1}{20}$



$$0 < \omega < \frac{1}{20}$$

$$\tilde{G}(s) = \frac{1,5}{s} \cdot \frac{s}{1} = \frac{1,5}{s}$$

$$|\tilde{G}(j\omega)| = \frac{1,5}{\omega} \qquad \measuredangle \tilde{G}(j\omega) = -90°$$

$$\omega = \frac{1}{20} \qquad |\tilde{G}(j\tfrac{1}{20})| = 1,5 \cdot 20 = 30$$

$R(s) = \mu_R$

$\omega_c \geq 5$ rad/s

$\varphi_m \geq 45°$

$\bar{\omega} = 25$ rad/s $\quad \mu_R > 0$

$\measuredangle \mu_R G(j\bar{\omega}) \geq -135°$

$\measuredangle G(j25) \geq -92°$

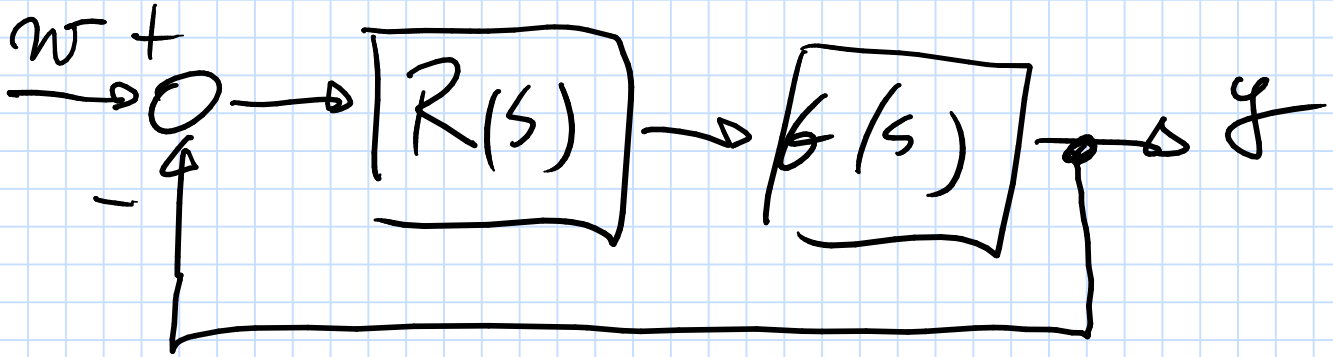
$\Downarrow$

$\varphi_m \geq 88°$ $\not$

$\mu_R \to |\mu_R G(j25)| = 1 \longrightarrow$

$$\frac{1{,}5\,\mu_R\,|1+j\cdot 75|}{25\,|1+j\cdot 500|} = 1$$

$$\mu_R \cong 3{,}330\cdot 10^{2}$$




$$G(s) = \frac{10}{s(1+s)}$$

(a) sovraelongazione nella risposta a scalino unitario

$S\% \leq 16\%$

(b) $F(s)$ presenti una coppia di poli c.c. dominante con $\omega_n = 2 \text{ rad/s}$

$\omega_n = 2 \Rightarrow \omega_c \cong \omega_n \Rightarrow \omega_c = 2$

coppia c.c. dominante $\Rightarrow \varphi_m < 75^\circ \Rightarrow \xi = \frac{\varphi_m}{100}$

$\xi < 0{,}75$

$\Delta\%_0 = 100\, e^{-\xi\pi/\sqrt{1-\xi^2}} \leq 16$

$$\xi = \frac{\ln\left(\frac{100}{\Delta\%_0}\right)}{\sqrt{\pi^2 + \left[\ln\frac{100}{\Delta\%_0}\right]^2}}$$

$\Delta_{\%_0} = 16 \quad \xi \approx 0{,}5$

$\Delta_{\%_0} \leq 16 \quad \xi \geq 0{,}5 \quad 50^\circ \leq \varphi_m \leq 75^\circ$

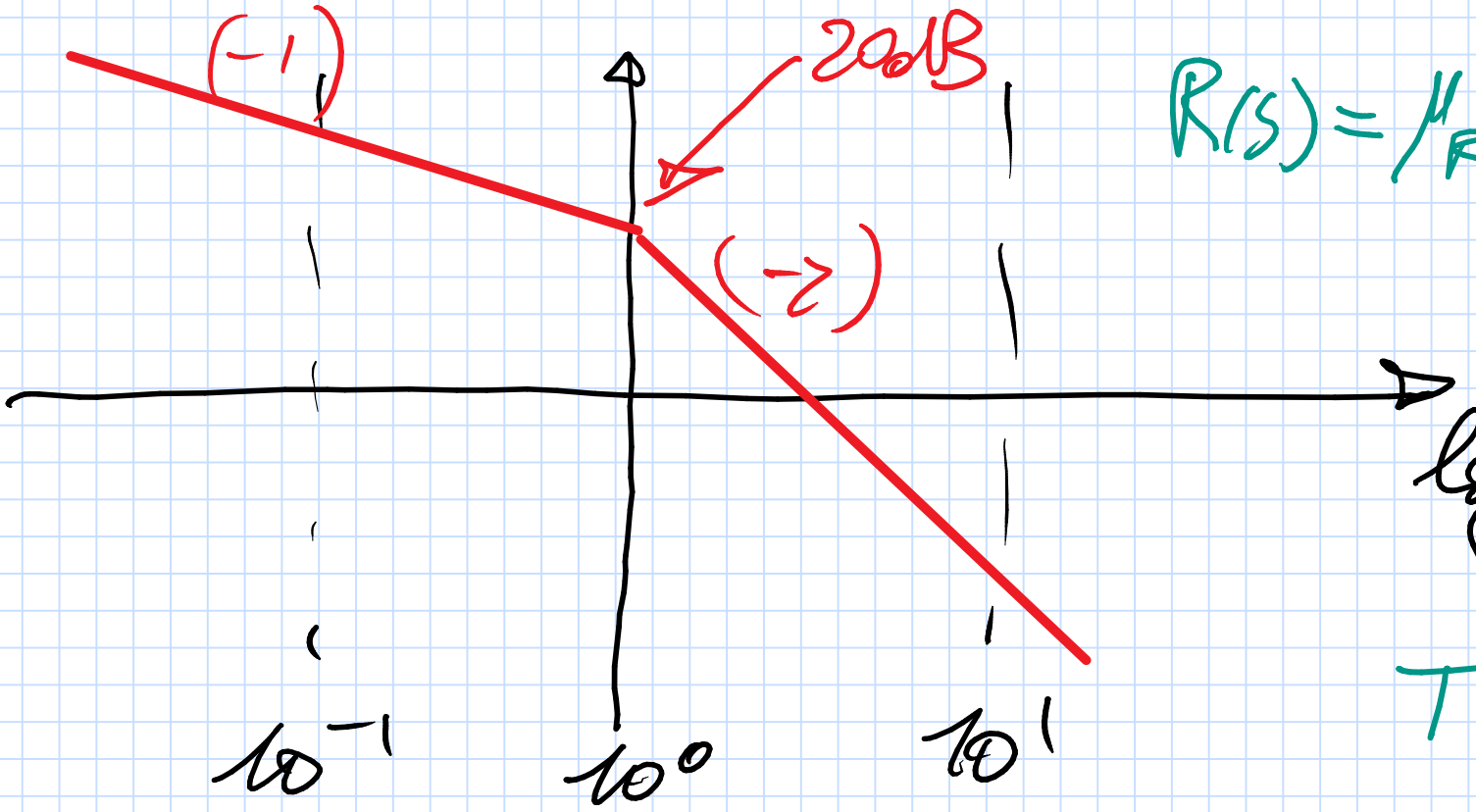
$\angle \mu_к G(j2) = ?$ $\qquad \mu_к > 0$

$\angle G(j2) = -90° - \arg(1+2j)$

$= -90° - \operatorname{arctg} 2 \approx -154°$

$\varphi_м \geq 26°$

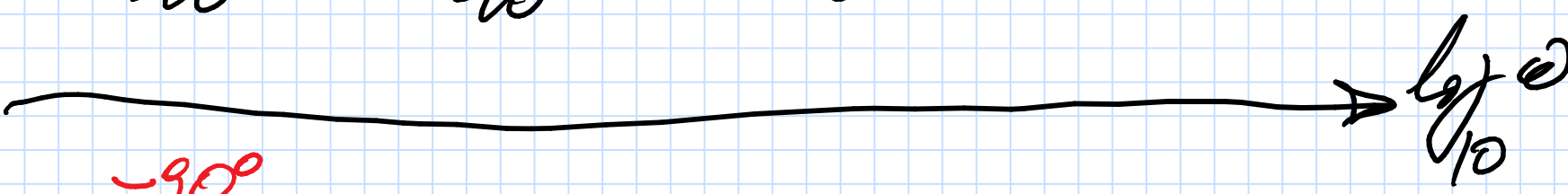
$50° \leq \varphi_м < 54°$




$$R(s) = \mu_R \frac{1+s}{1+Ts}$$

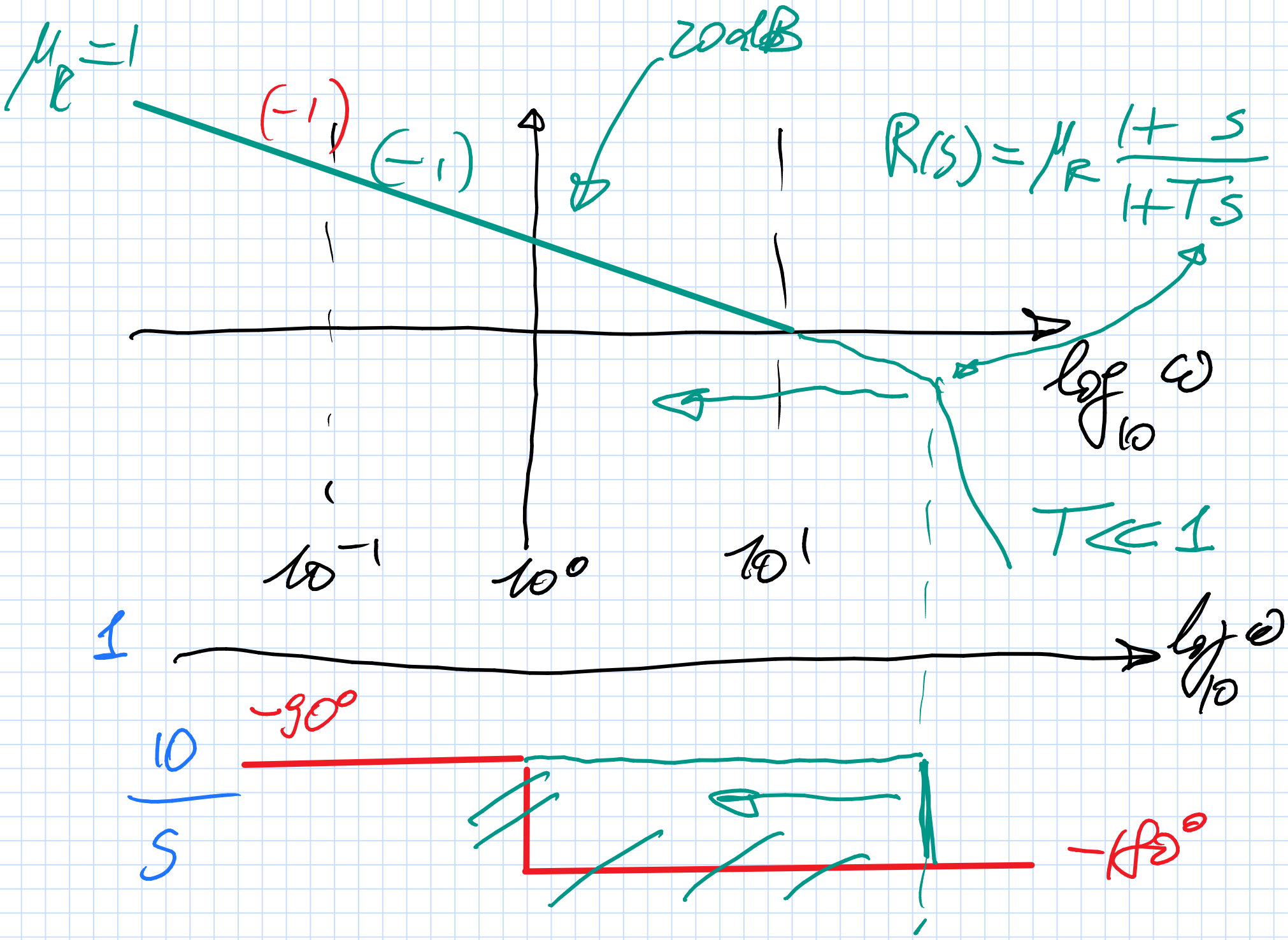
$T \ll 1$

$0 < \omega < 1$




$$\tilde{G}(s) = \frac{10}{s}$$




$\mu_R = 1$
20dB
(-1)
(-1)
$R(s) = \mu_R \frac{1+s}{1+Ts}$
$\log_{10} \omega$
$10^{-1}$
$10^{0}$
$10^{1}$
$T \ll 1$
1
$\log_{10} \omega$
-90°
$\frac{10}{s}$
-45°

$$L(s) = \frac{10 \mu_R (1+s)}{s(1+Ts)(1+s)}$$

$$\sphericalangle L(j2) = -120^\circ$$

$$= 0^\circ - 90^\circ - \operatorname{arctg}(2T) = -120^\circ$$

$$+\operatorname{arctg}(2T) = +30^\circ$$

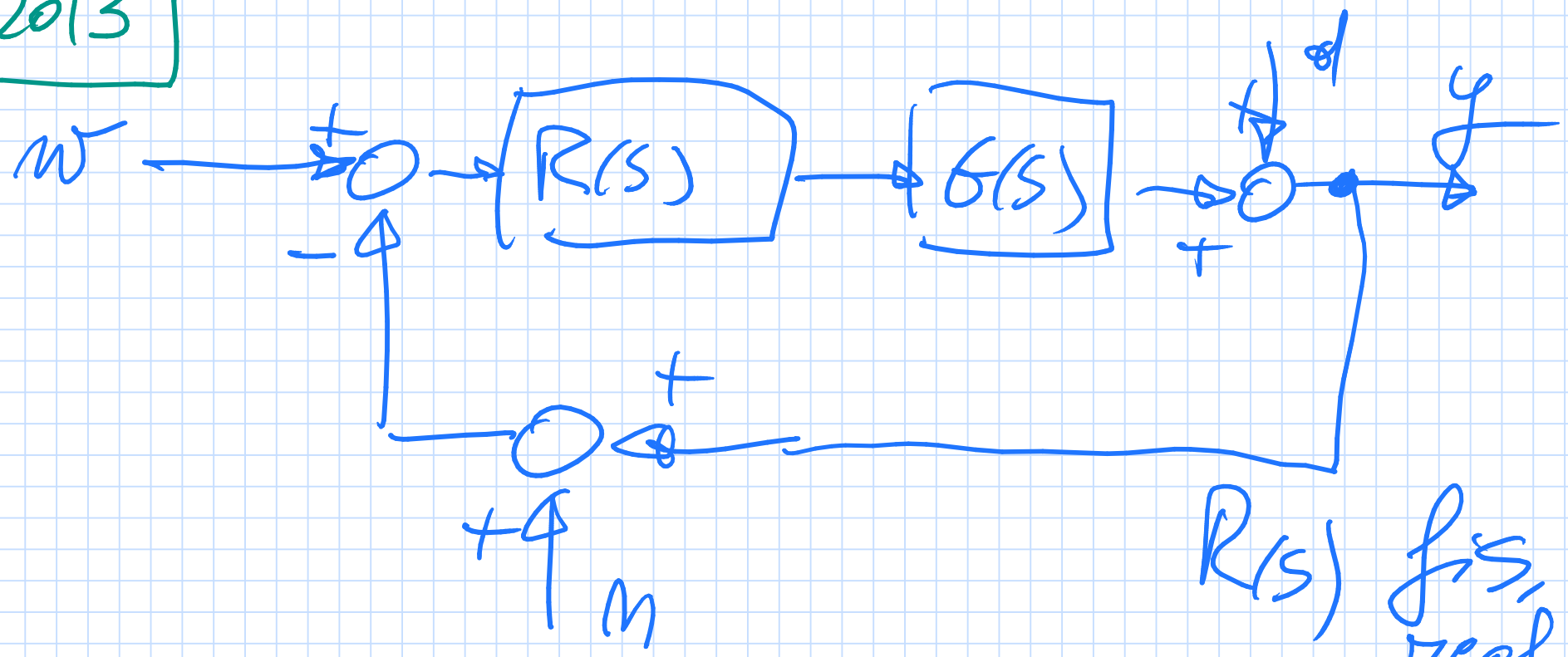
$$2T = \operatorname{tg} 30^\circ$$

$$T = \frac{1}{2}\left[\operatorname{tg} 30^\circ\right]$$

$$|L(j\omega)| = 1 \qquad \frac{10\mu_R}{2|1+2j\tau|} = 1$$

$$\mu_R = \cdots$$

9/9/2013




$$G(s) = 10 \cdot \frac{1+5s}{1+0{,}4s+s^2}$$

(a) $e_\infty \to 0$ $\quad w(t) = A \cdot 1(t)$ $\quad n(t) \equiv 0$

$d(t) = B \cdot 1(t)$ $\quad A, B \in \mathbb{R}$

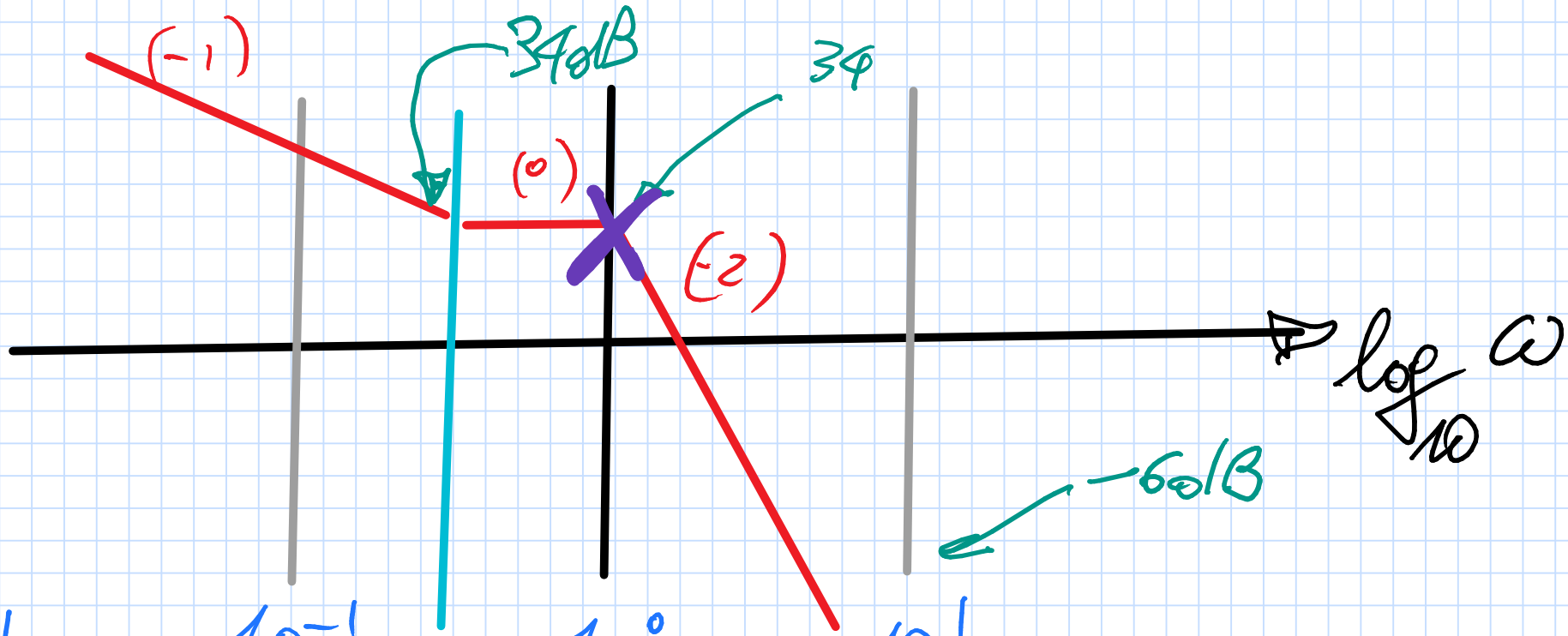
(b) $5 \le \omega_c \le 10$ $\quad$ (c) $\varphi_M \ge 60°$

(a) → $R_1(s) = \frac{\mu_R}{s}$

$$L_1(s) = \frac{10\mu_R(1+5s)}{s(1+0{,}4s+s^2)}$$

$P_1 = 0$ $\quad$ $\Sigma_1$ $\begin{cases} \omega_n = 1 \\ \xi = 0{,}2 \end{cases}$ $\quad$ $\delta_1 \rightarrow 1/5 = 0{,}2$

$|\cdot|$

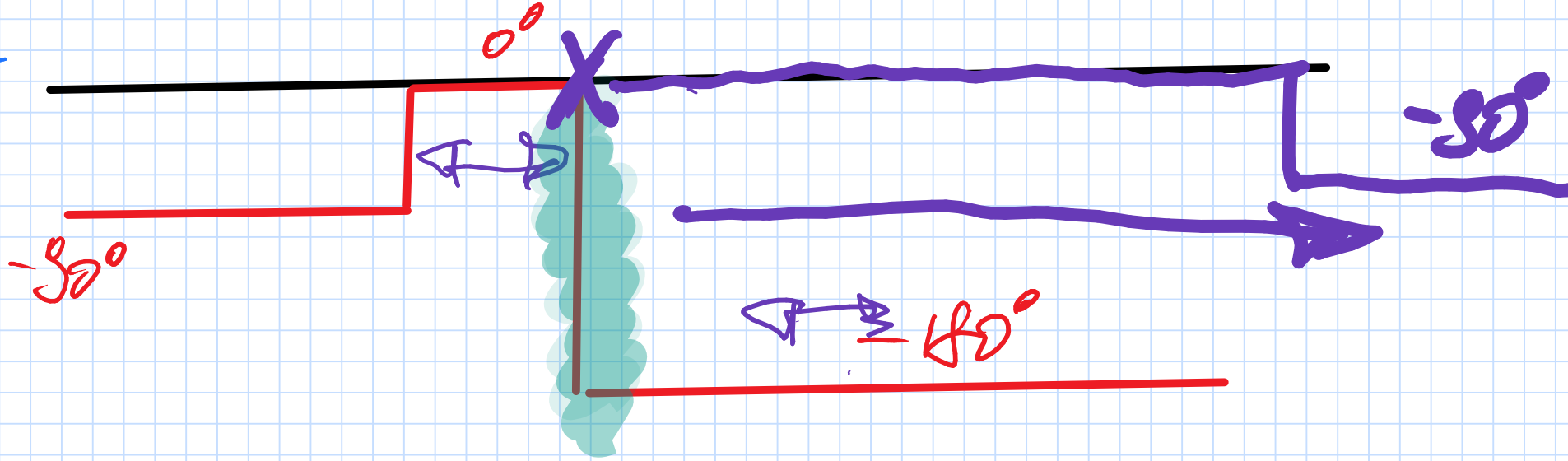



$\mu_R = 1$

$0 \leq \omega < \frac{1}{5}$

$\omega = 1/5$

$\frac{10 \cdot 1}{s \cdot 1}$



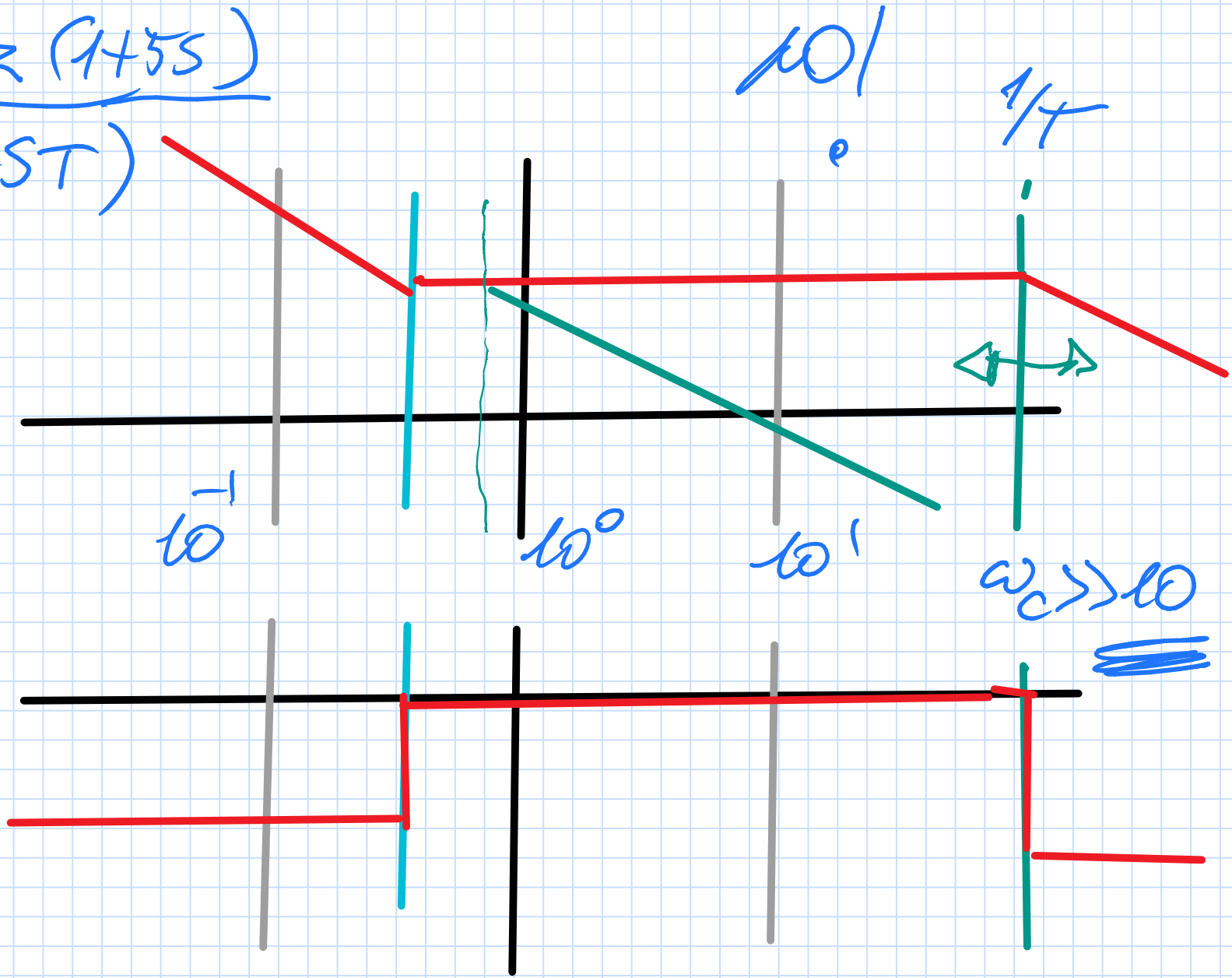
$$R_2(s) = \frac{1 + 0{,}4s + s^2}{1 + sT} \qquad R_2 = \frac{(1+s)^2}{1+sT}$$

$$R(s) = \mu_R \cdot \frac{1 + 0{,}4s + s^2}{s(1 + sT)}$$

$$R(s) = \mu_R \frac{(1+s)^2}{s(1+sT)}$$

PID

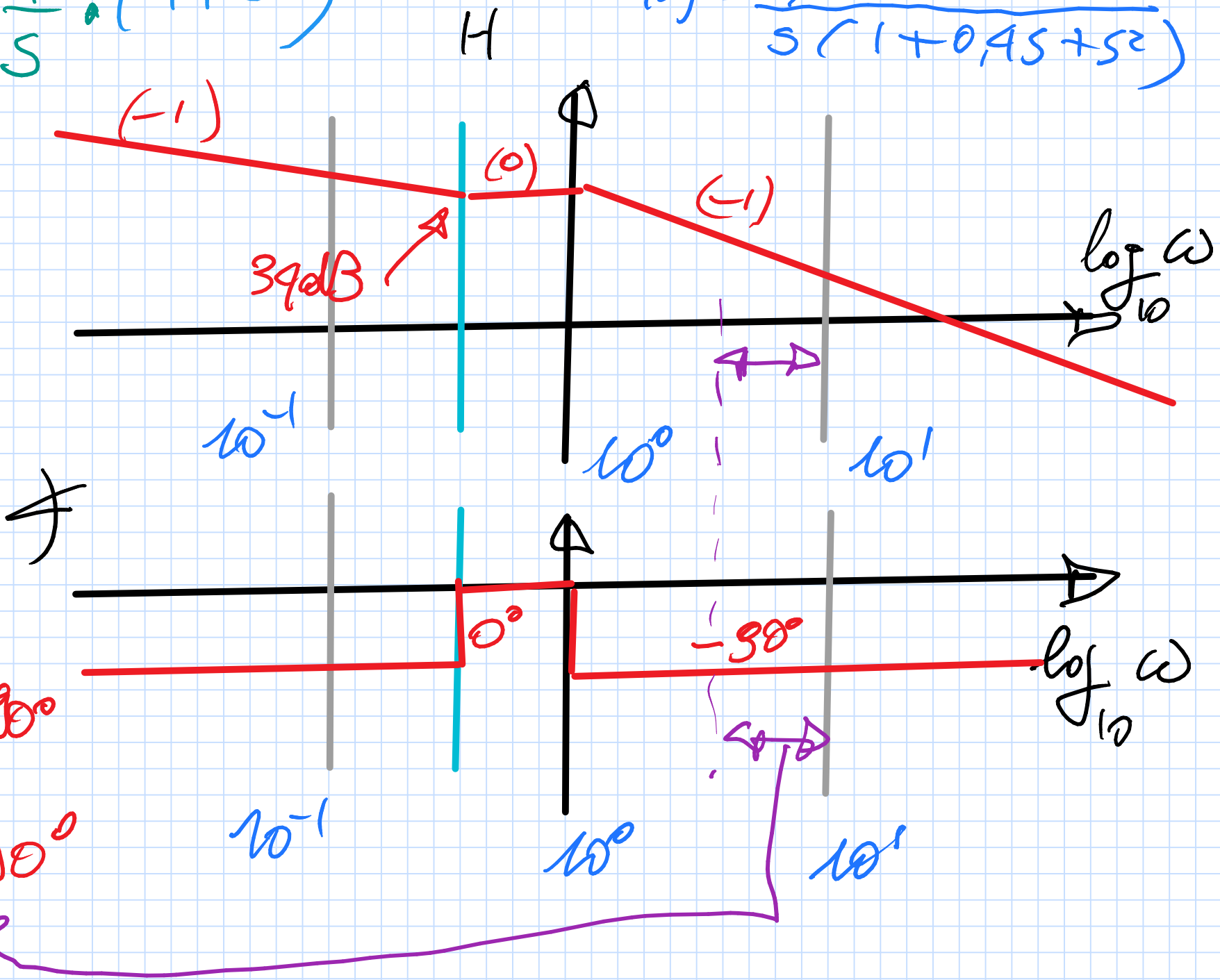
$$L(s) = \frac{10 K_R (1+5s)}{s(1+sT)}$$



$$R(s) = \mu_R \frac{1}{s} \cdot (1+s)$$

$$L(s) = \frac{10\mu_R(1+s)(1+s)}{s(1+0{,}4s+s^2)}$$

PI




$\varphi_m \geq 90°$

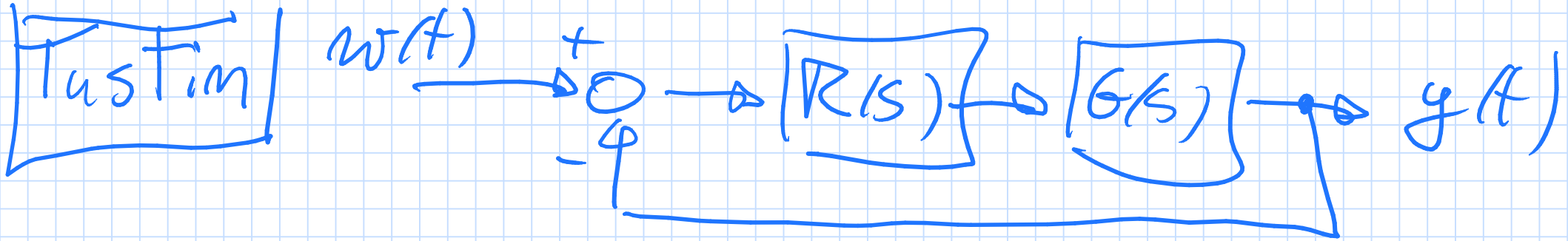
$\omega_c = 7$

$$|L(j7)| = 1$$

$$\frac{10}{7}\mu_R \frac{|(1+35j)(1+7j)|}{|1+2,8j-98|} = 1$$

$$\mu_R \geq 0,136$$

$$\angle L(j7) \geq -96^\circ \qquad \varphi_m \geq 84^\circ$$




$$R(s) = 25 \cdot \frac{1}{s(1+0{,}5s)}$$

$\omega_c \geq 2{,}3$ rad/s

$\varphi_M \geq +4°$

$\Delta = ?$, $R_w(t) = ?$

$|\delta_\varphi| \leq 4°$

$$|\delta_\varphi| = \left| -\frac{\omega\Delta}{2} \cdot \frac{180}{\pi} \right|$$

$$|\delta_\varphi| = \frac{\omega_c \Delta}{2} \cdot \frac{180}{\pi} \leq 4$$

$\omega_c = 2{,}3$

$$\Delta \leq \frac{8\pi}{180\omega_c} \geq 6 \cdot 10^{-2}\ s$$

$\Delta = 5 \cdot 10^{-2}$

$1 \cdot 10^{-2}$

$$s = \frac{2}{\Delta}\,\frac{z-1}{z+1}$$

$$R_w(z) = 25 \cdot \frac{1}{\frac{2}{\Delta}\,\frac{z-1}{z+1}\left(1 + 0{,}5 \cdot \frac{2}{\Delta}\,\frac{z-1}{z+1}\right)} \Rightarrow$$

$$R_{TO}(z) = \frac{25\Delta^2}{2} \cdot \frac{(z+1)^2}{(z-1)\left[(\Delta+1)z + (\Delta-1)\right]}$$

zeri del denominatore

polo $s = 0$

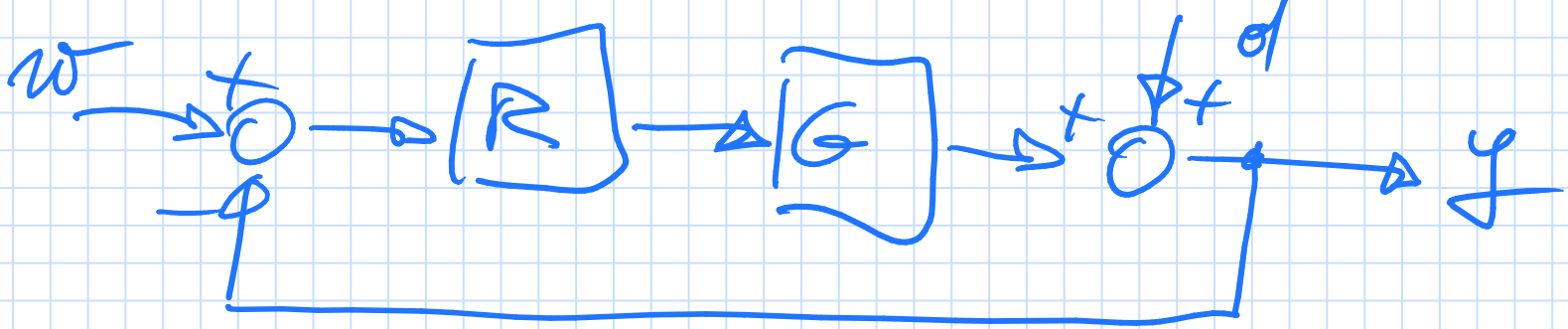
polo $m = -2$

$s = 0$

$s = \frac{2}{\Delta}\frac{z-1}{z+1}$

$s = -2$

18/7/14




$$G(s) = \frac{10}{1+10s} e^{-2s}$$

(a) $e_\infty \to 0$ per $w(t) = A \cdot 1(t)$
$d(t) = B \cdot 1(t)$ $\quad A, B \in \mathbb{R}$

(b) $d = 0 \qquad \omega_c \geq 0{,}5 \text{ rad/s}$

(c) $d = 0 \qquad \varphi_m > 30°$

$$G_1(s) = \frac{10}{1+10s}$$

$$G(s) = G_1(s)\, e^{-2s}$$

$\bar{\omega}$ come "pulsazione che $\omega_c$ candida"

(a) identica

(b) identica

(c) ← modifica!

$$\angle G(j\bar{\omega}) = \angle G_1(j\bar{\omega}) + \angle e^{-2j\bar{\omega}}$$

$$\angle G(j\tilde{\omega}) = \angle G_1(j\tilde{\omega}) - 2\tilde{\omega}\cdot\frac{180}{\pi}$$

$$\varphi_c = \varphi_c^{(1)} - \underbrace{\left[2\tilde{\omega}\,\frac{180}{\pi}\right]}_{\Delta\varphi}$$

$$\varphi_c = \varphi_c' - \Delta\varphi$$

$$\varphi_m = 180^\circ - |\varphi_c| \geq 30^\circ$$

$$\varphi_m' = ?$$

$$\varphi_m' \geq 30^\circ + \Delta\varphi$$

$$G_1(s) = \frac{10}{1+10s}$$

$\bar{\omega} \neq \omega_c$

(a), (b), $\varphi_m^{(1)} \geq 30° + 2\bar{\omega} \cdot \frac{180}{\pi}$

(a) $R_1(s) = \frac{\mu_R}{s}$ $\quad R_2(s) = (1+10s)$

$$R(s) = \mu_R \frac{1+10s}{s} \quad (PI)$$

$$L_1(s) = \frac{10\mu_R}{s}$$

$$L_2(s) = \frac{10\mu_R}{s} e^{-2s}$$

$$\underbrace{-\frac{\pi}{2}}_{\measuredangle L_1} \underbrace{-2\bar{\omega}}_{e^{-2s}} \geq -\frac{5}{6}\pi \qquad (-150^\circ)$$

$$2\bar{\omega} \leq \frac{\pi}{3}$$

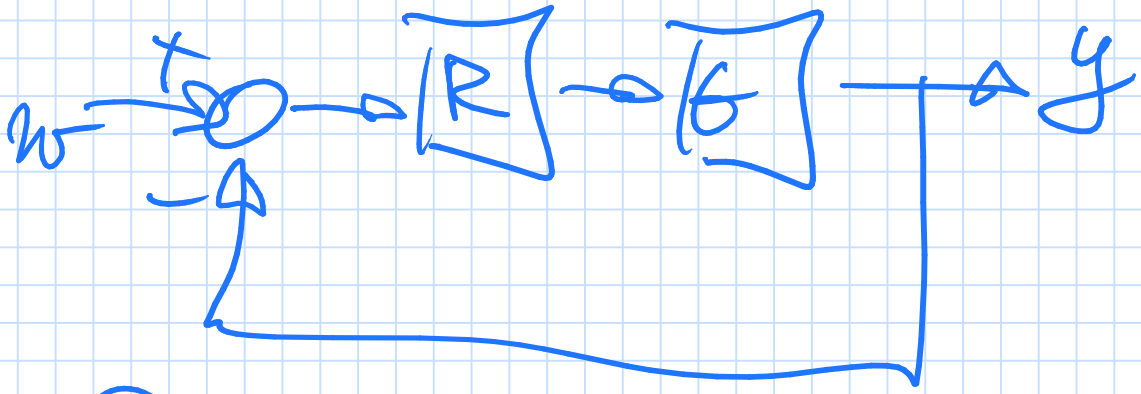
$$\bar{\omega} \leq \frac{\pi}{6} \approx 0{,}5236\ldots$$

$$0{,}5 \leq \omega_c \leq \frac{\pi}{6} \qquad \bar{\omega}_c = 0{,}52$$

$\bar{\omega} = 0{,}52 \longrightarrow \varphi_C \approx -149^\circ 32'$

$\varphi_M \approx 30^\circ 27'$

# Progetto per sistema SDV a fase minima




$$G(s) = 4\,\frac{(1-0{,}5s)}{(1+s)(1+0{,}1s)}$$

(a) $e_\infty \to 0$

(b) $\varphi_m > 30°$

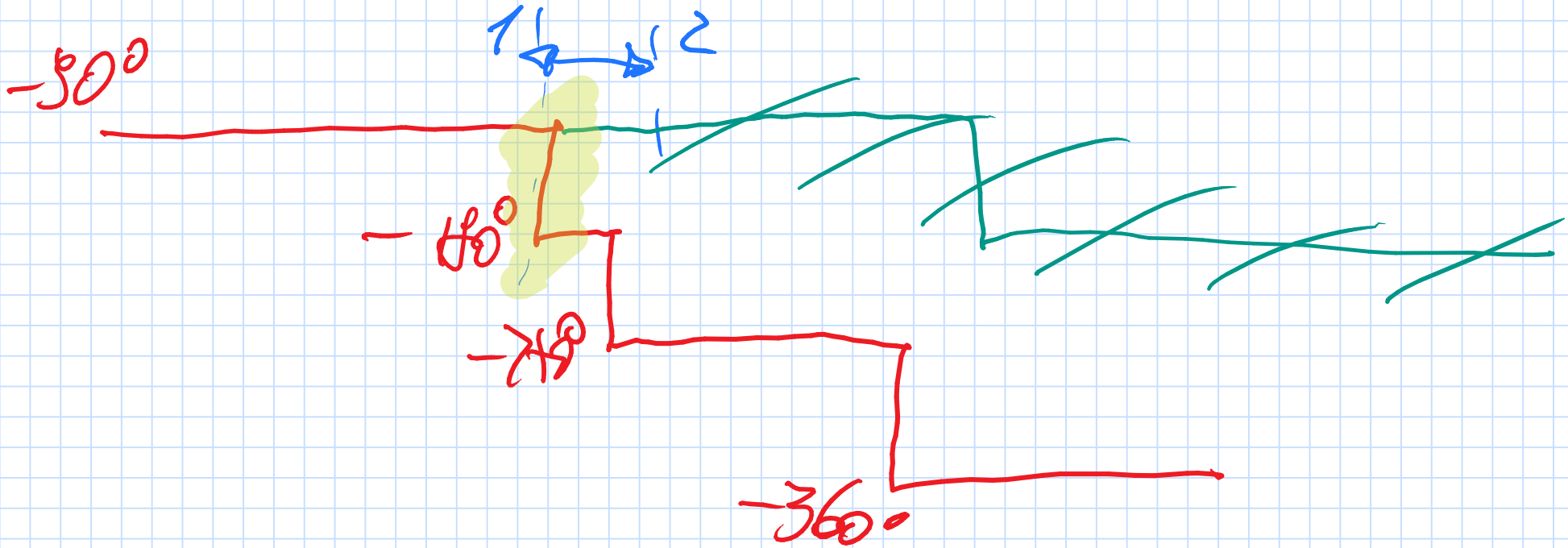
(c) $\omega_c > 1$ rad/s

(a) → $R_1(s) = \frac{\mu_R}{s}$

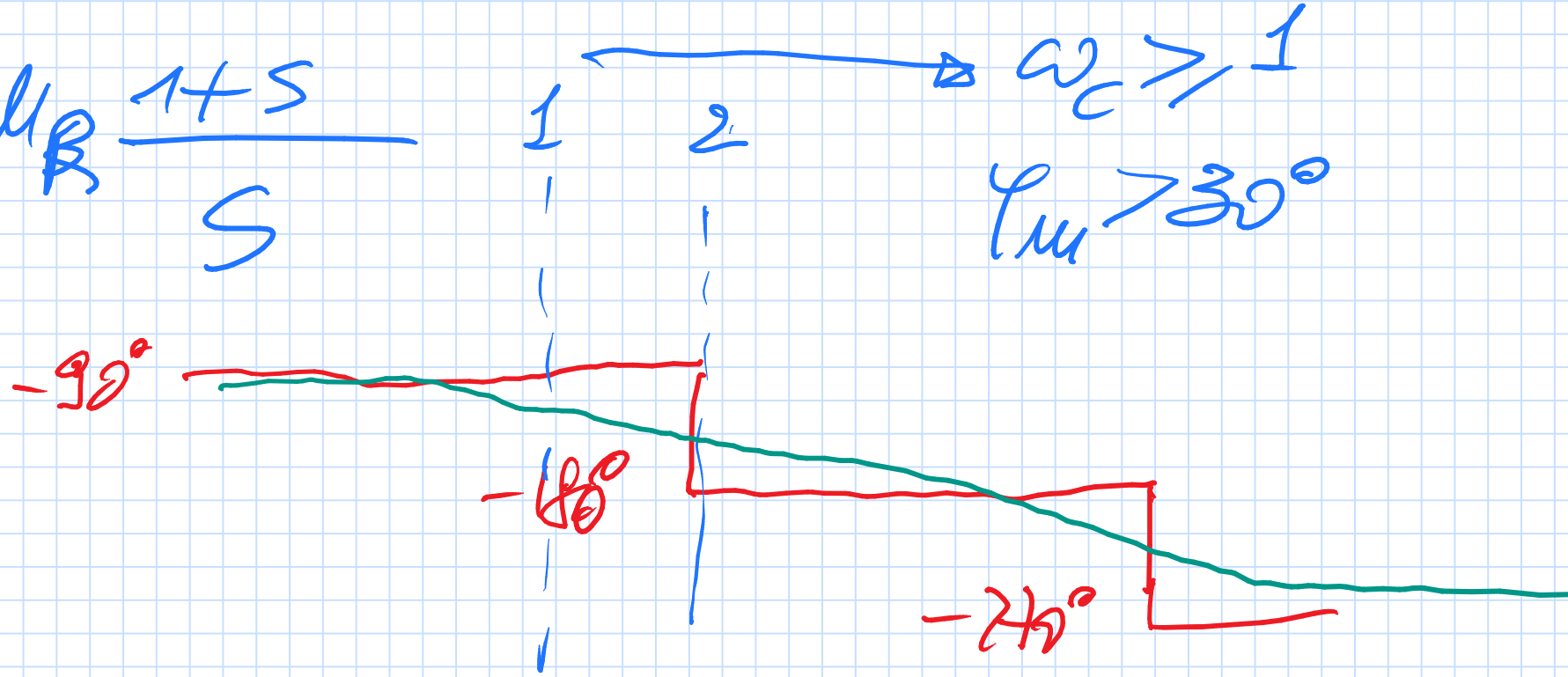
$$L_1(s) = \frac{4\mu_R(1-0{,}5s)}{s(1+s)\left(1+\frac{s}{10}\right)}$$

$1 < \omega < 10$ → $-180° \rightarrow -360°$

$1 < \omega < 2$ → $-180° \rightarrow -270°$




$$R(s) = \mu_R \frac{1+s}{s}$$



$$L_2(s) = 4\mu \frac{\left(1 - \frac{s}{2}\right)}{s\left(1 + \frac{s}{10}\right)}$$

$$\angle L_2(j\omega) > -150^\circ \qquad (\varphi_m > 30^\circ)$$

$$-\operatorname{arctg}\frac{\omega}{2} - 90^\circ - \operatorname{arctg}\frac{\omega}{10} > -150^\circ$$

$$\operatorname{arctg}\left(\frac{\omega}{2}\right) + \operatorname{arctg}\left(\frac{\omega}{10}\right) < 60^\circ$$

$$\operatorname{arctg}\left(\frac{\omega}{2}\right)+\operatorname{arctg}\left(\frac{\omega}{10}\right)<60^\circ$$

$$\operatorname{tg}(\alpha+\beta)=\frac{\operatorname{tg}\alpha+\operatorname{tg}\beta}{1-\operatorname{tg}\alpha\cdot\operatorname{tg}\beta}$$

$$\frac{\frac{\omega}{2}+\frac{\omega}{10}}{1-\frac{\omega^2}{20}}<\sqrt{3}$$

$1-\frac{\omega^2}{20}>0$

$$1-\frac{\omega^2}{20}>0 \Rightarrow \omega<\sqrt{20}$$

$$\frac{\sqrt{3}}{20}\omega^2 + \frac{6}{10}\omega - \sqrt{3} < 0$$

$$\begin{cases} -9,12 < \omega < 2,19 \\ 0 < \omega < 4,4 \end{cases}$$

$\sqrt{20}$

$$0 < \omega < 2,19$$

$$\omega_c \geq 1$$

per es $\omega_c = 1,5 \Rightarrow M_R \cong 0,3$

$\varphi_m \geq 45^\circ$