Adaptive Fault-Tolerant Control of Nonlinear Uncertain Systems: An Information-Based Diagnostic Approach

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***Abstract*—This paper presents a unified methodology for detecting, isolating and accommodating faults in a class of nonlinear dynamic systems. A fault diagnosis component is used for fault detection and isolation. On the basis of the fault information obtained by the fault-diagnosis procedure, a fault-tolerant control component is designed to compensate for the effects of faults. In the presence of a fault, a nominal controller guarantees the boundedness of all the system signals until the fault is detected. Then the controller is reconfigured after fault detection and also after fault isolation, to improve the control performance by using the fault information generated by the diagnosis module. Under certain assumptions, the stability of the closed-loop system is rigorously investigated. It is shown that the system signals remain bounded and the output tracking error converges to a neighborhood of zero.**

***Index Terms*—Fault detection and isolation, fault-tolerant control, neural networks, nonlinear systems.**

I. Introduction

THE greater demand for productivity has led to more challenging operating conditions for many modern industrial systems. Such conditions increase the possibility of system faults, which are characterized by critical and unpredictable changes in system dynamics. A fault-tolerant control (FTC) system is capable both of automatically compensating for the effects of faults and of maintaining the performance of the controlled system, at some acceptable level, even in the presence of faults. In general, fault tolerance can be achieved either passively by use of feedback control laws that are robust to possible system faults, or actively by means of a fault diagnosis [fault detection and isolation (FDI)] and accommodation architecture. Survey papers by Patton [17] and Isermann *et al.* [11] provide excellent overviews of recent research work on FTC.

Over the last two decades, the design and analysis of fault-diagnosis algorithms using the model-based analytical redundancy approach have received significant attention [3], [8], [9]. The fault information generated by detection and isolation procedures can be very useful to FTC. However, links between fault diagnosis and FTC techniques are still lacking [17]. Some recent results on the integration of FDI with FTC can be found in [12], [25], [27], and [30].

This paper presents a unified design and analysis methodology for detecting, isolating, and accommodating faults in a class of nonlinear dynamic systems. The proposed fault-diagnosis and accommodation scheme has two main components: 1) *the online health monitoring (fault diagnosis) module* consists of a bank of nonlinear adaptive estimators. One of them is the fault detection and approximation estimator (FDAE), while the others are fault isolation estimators (FIEs); and 2) *the controller (fault accommodation) module* consists of a nominal controller and two fault-tolerant controllers, which are used right after fault detection and isolation, respectively.

To facilitate the design of nonlinear fault-tolerant controllers, we consider a class of single-input–single-output nonlinear systems in a triangular structure, subject to unstructured (possibly nonlinear) modeling uncertainty and nonlinear faults. A fault is assumed to be an unknown nonlinear function of the state of a closed-loop system. Under normal operating conditions (without faults), the nominal controller aims to guarantee the stability of the closed-loop system, while the FDAE is used to monitor the system to detect the occurrence of any fault. In the presence of a fault, under certain assumptions the nominal controller is shown to maintain the system signal boundedness until the fault is detected by the FDAE. After fault detection, the nominal controller is reconfigured to compensate for the effect of the fault, and at the same time the bank of FIEs is activated to establish the particular type of fault that has occurred. If the fault is isolated on the basis of the fault information obtained by the isolation procedure, the second fault-tolerant controller is activated to try to enhance the control performance. The overall architecture adopts a learning-based approach by which an unknown fault is estimated online by using adaptive and online approximation techniques. Alternative adaptive methods for fault diagnosis and accommodation can be found in [2], [4], [7], [12], [21], and [23].

Prior papers by the authors deal with the FDI properties [20], [29]. Here, we consider the problem of integrating the FDI scheme with a fault-accommodation design. First, the closed-loop system's stability in the presence of a fault, but before its detection, is investigated. Then, in order to compensate for the effect of the fault, two fault-tolerant controllers are designed that are used after fault detection but before its isolation and after fault isolation, respectively. It is shown that all the system signals remain bounded and the output tracking error

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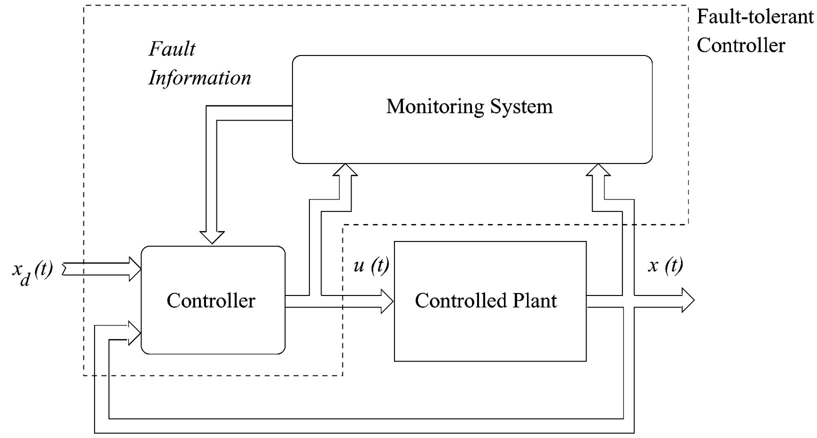

Fig. 1. Architecture of the fault-tolerant control scheme.

converges to a neighborhood of zero. Moreover, the second fault-tolerant controller, which is designed on the basis of the fault information obtained by the isolation procedure, requires fewer assumptions than the first fault-tolerant controller.

The paper is organized as follows. In Section II, the problem of fault-tolerant control of nonlinear uncertain systems is formulated. In Section III, the FDI module is briefly described, and in Section IV, the design and analysis of the controller module are addressed. To illustrate the proposed FTC methodology, a simulation example is given in Section V. Finally, some concluding remarks are made in Section VI.

II. Problem Formulation

This paper presents a fault-tolerant nonlinear control architecture, as shown in Fig. 1.

The fault-tolerant controller will be designed according to the following qualitative objectives.

1) In a fault-free mode of operation, the state $x(t)$ should track the reference vector $x_d(t)$ as closely as possible, even in the possible presence of plant modeling uncertainty.
2) When a fault occurs, the controller should be able to guarantee some stability property, such as boundedness of all signals in a closed-loop system.
3) The control action $u(t)$ generated by the controller should accommodate the fault that has occurred and recover the tracking performances by using the fault information provided by the *monitoring module*.

It is worth noting that the choice of using the monitoring and controller modules as two separate subsystems may be rather suitable for practical implementation because, in a way, it reflects the typical hierachical structure of modern integrated automation systems.

A. Controlled Plant

We consider a general multivariable nonlinear dynamic system described by

$$\dot{x} = \phi(x,u) + \eta(x,u,t) + \mathcal{B}(t-T_0)f(x,u) \tag{1}$$

where $x \in \Re^n$ is the state vector of the system, $u \in \Re^m$ is the input vector, $\phi, f : \Re^n \times \Re^m \mapsto \Re^n$, and $\eta : \Re^n \times \Re^m \times \Re^+ \mapsto \Re^n$ are smooth vector fields, and $\mathcal{B}(t-T_0)$ is a matrix function representing the time profiles of faults, where T_0 denotes the unknown fault-occurrence time. The vector fields ϕ, η, and f denote the dynamics of the nominal model, the modeling uncertainty, and the change in the system dynamics due to a fault, respectively.

The modeling uncertainty, represented by the vector field η, includes external disturbances as well as modeling errors. The following assumption will be used.

Assumption 1: The modeling uncertainty, denoted by η in (1), is an unknown nonlinear function of x, u, and t, but bounded by some known function $\bar{\eta}$. Specifically, each component η_i of the modeling error η is assumed to satisfy

$$|\eta_i(x,u,t)| \leq \bar{\eta}_i(x,u,t) \quad \forall x \in \Re^n \quad \forall u \in \Re^m \quad \forall t \in \Re^+ \tag{2}$$

where the bounding function $\bar{\eta}_i(x,u,t) \geq 0$, for $i = 1,\ldots,n$ is known, continuous and uniformly bounded.

As will be seen later on, the uniform boundedness assumption on $\bar{\eta}_i(x,u,t)$ is simply a technical existence condition needed for the theoretical analysis to guarantee the boundedness of system signals before fault detection. Moreover, it is important to emphasize that, if we allow each $\bar{\eta}_i$ to be a function of x, u, and t, the above formulation provides a framework for *nonuniform bounds*, thus enhancing the achievable fault sensitivity and decreasing the detection and isolation times (both the FDI and the controller modules will be designed on the basis of the nonuniform bound $\bar{\eta}_i(x,u,t)$). For example, in several applications, the nominal model is obtained by small-signal linearization techniques (around a nominal operating point, or trajectory). In this case, $\eta(x,u,t)$ may represent the residual nonlinear terms, which are typically small for (x,u) close to the operating point but can be large elsewhere.

From a qualitative viewpoint, the term $\mathcal{B}(t-T_0)f(x,u)$ represents the deviation in the system dynamics due to the occurrence of a fault. The matrix $\mathcal{B}(t-T_0)$ characterizes the time profile of a fault that occurs at some *unknown* time T_0, and $f(x,u)$ denotes the nonlinear fault function. This characterization allows both additive and multiplicative faults

[9], as well as more general nonlinear faults. We let the fault time profile $\mathcal{B}(\cdot)$ be a diagonal matrix of the form

$$\mathcal{B}(t-T_0) \triangleq \mathrm{diag}[\beta_1(t-T_0),\ldots,\beta_n(t-T_0)]$$

where $\beta_i : \Re \mapsto \Re$ is a function representing the time profile of a fault affecting the ith state equation, for $i = 1,\ldots,n$. We consider faults with time profiles modeled by

$$\beta_i(t-T_0) = \begin{cases} 0, & \text{if } t < T_0 \\ 1 - e^{-a_i(t-T_0)}, & \text{if } t \geq T_0 \end{cases} \tag{3}$$

where the scalar $a_i > 0$ denotes the unknown fault-evolution rate. Small values of a_i characterize slowly developing faults, also known as *incipient faults*. For large values of a_i, the time profile β_i approaches a step function that models *abrupt faults*. Note that the fault-time profile given by (3) stands for only the developing speed of a fault, whereas all its other basic features are defined by the nonlinear function $f(x,u)$ described later.

Since the fault-tolerant controller proposed in the paper makes explicit use of information about the fault that has occurred, i.e., the detection and isolation of the fault provided by the monitoring module (see Fig. 1), we assume that there are N types of possible nonlinear fault functions; specifically, $f(x,u)$ belongs to a finite set of functions

$$\mathcal{F} \triangleq \{f^1(x,u),\ldots,f^N(x,u)\}.$$

Each fault function $f^s, s = 1,\ldots,N$, is described by

$$f^s(x,u) \triangleq \left[(\theta_1^s)^\top g_1^s(x,u),\ldots,(\theta_n^s)^\top g_n^s(x,u)\right]^\top$$

where $\theta_i^s, i = 1,\ldots,n$, is an unknown q_i^s-dimensional parameter vector assumed to belong to a known compact set Θ_i^s (i.e., $\theta_i^s \in \Theta_i^s \subset \Re^{q_i^s}$) and g_i^s: $\Re^n \times \Re^m \mapsto \Re^{q_i^s}$ is a known smooth vector field. This representation characterizes a general class of faults where the nonlinear vector field g_i^s represents the functional structure of the sth fault affecting the ith state equation, whereas the unknown parameter vector θ_i^s characterizes the "magnitude" of the fault in the ith state equation. The dimension q_i^s of each parameter vector θ_i^s is determined by both the type of fault and the specific state component considered.

B. Fault-Tolerant Controller

First of all, we define three important time-instants: T_0 is the fault-occurrence time; $T_d > T_0$ is the fault-detection time; $T_{\mathrm{isol}} > T_d$ is the fault-isolation time when the monitoring system determines (if possible) which fault of the class $\mathcal{F}$ has actually occurred. The structure of the fault-tolerant controller is given by

$$\dot{v} = \begin{cases} g_0(v,x,x_d,t) & \text{for } t < T_d \\ g_D(v,x,x_d,t), & \text{for } T_d \leq t < T_{\mathrm{isol}} \\ g_I(v,x,x_d,t), & \text{for } t \geq T_{\mathrm{isol}} \end{cases}$$

$$u = \begin{cases} h_0(v,x_d,t), & \text{for } t < T_d \\ h_D(v,x_d,t), & \text{for } T_d \leq t < T_{\mathrm{isol}} \\ h_I(v,x_d,t), & \text{for } t \geq T_{\mathrm{isol}} \end{cases} \tag{4}$$

where $v \in \Re^r$ is the state vector of the controller and $x_d \in \Re^n$ denotes a reference vector to be tracked by the controlled system state vector; $g_0, g_D, g_I : \Re^r \times \Re^n \times \Re^n \times \Re^+ \mapsto \Re^r$ and $h_0, h_D, h_I : \Re^r \times \Re^n \times \Re^+ \mapsto \Re^m$ are nonlinear functions to be designed according to the following objectives.

- Under normal operating conditions (i.e., for $t < T_0$), a nominal controller g_0, h_0 is designed to guarantee the system's stability and robust tracking performance in the presence of the modeling uncertainty η.
- When a fault occurs at time T_0, the nominal controller described by g_0 and h_0 should guarantee the system signal boundedness until the fault is detected, i.e., for $t < T_d$.
- After fault detection (i.e., for $t \geq T_d$), the nominal controller is *reconfigured* to compensate for the effect of the (yet unknown) fault, that is, the controller described by g_D and h_D is designed in such a way as to exploit the information that a fault has occurred so that the controller may recover some control performances (e.g., tracking of $x_d(t)$). This new controller should guarantee the boundedness of system signals even in the presence of the fault.
- If the fault is isolated (i.e., for $t \geq T_{\mathrm{isol}} \geq T_d$), then the controller is reconfigured again. The functions g_I and h_I are designed using the information about the type of fault that has actually occurred so as to improve the control performances.

Remark 1: It is possible that, in some cases, the fault that has occurred cannot be isolated, for instance a fault whose functional structure is completely unknown *a priori* (i.e., $f(x,u)$ does not belong to $\mathcal{F}$). Then, the first fault-tolerant controller guarantees some minimal performance (e.g., closed-loop stability). In this case, the second fault-tolerant controller cannot be activated.

Remark 2: As compared with some recent work on fault-tolerant control [7], [2], [12], [19], [30], [25]–[27], [4], [21], the previous framework allows us to make a thorough analysis of the issues involved in integrating fault diagnosis with FTC. Our approach explicitly takes into account the effects of fault-detection time and fault-isolation time on system stability and control performance. Moreover, some other issues, such as the occurrence of an unanticipated fault with an unknown functional structure, incipient faults, and unstructured modeling uncertainty, are considered as well.

III. Monitoring Module

A more detailed architecture of the fault-tolerant controller scheme is shown in Fig. 2. The controller module includes both an accommodation scheme determining which of the three controllers in (4) is activated at a specific time-instant and the design of reconfigurable controllers (see Section IV).

The monitoring module consists of a bank of $N+1$ nonlinear adaptive estimators operating in parallel. One of the adaptive estimators is the *fault detection and approximation estimator* (FDAE) used to detect and approximate faults. The remaining adaptive estimators are *fault isolation estimators* (FIEs) activated for the purpose of fault isolation only after a fault has been detected.

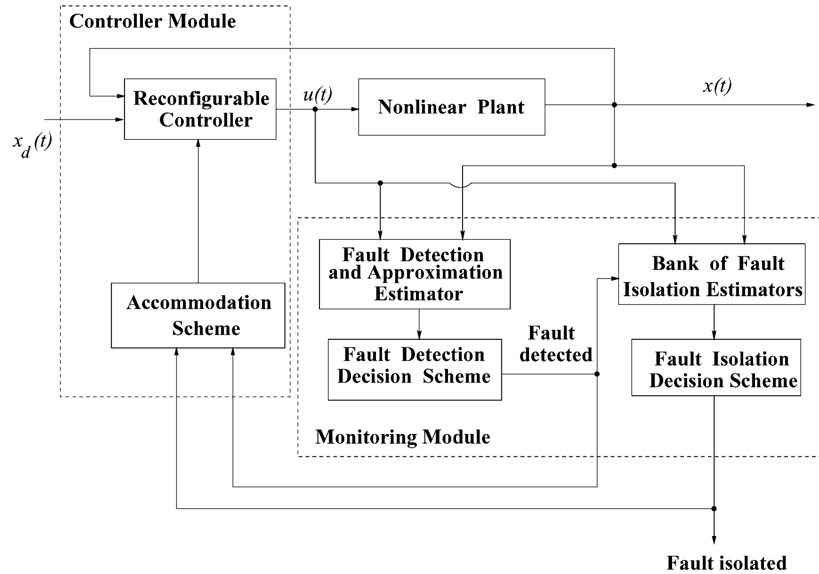

Fig. 2. Detailed architecture of the fault-tolerant control scheme.

Remark 3: Using the FDAE and the FIEs separately has two advantages. 1) Since the fault-isolation scheme consists of a bank of N estimators, there is no need to use them for fault detection. Hence, a single estimator is used for fault detection, whereas the bank of FIEs is activated only after fault detection. 2) In the presence of an unanticipated and completely unknown fault, the functional approximator included in the FDAE provides the adaptive structure for approximating on-line the unknown fault function (see Section III-A). This estimated fault model can be used in a subsequent phase (for instance, by a maintenance procedure) to improve the fault detection and isolation scheme by updating the fault class $\mathcal{F}$ and, accordingly, the bank of isolation estimators. Moreover, in some cases (e.g., the occurrence of an unanticipated fault or when two or more faults are indistinguishable), the fault that has occurred cannot be isolated online, but it is however important to make a detection decision in order to activate the first fault-tolerant controller, which takes care of the system stability (see Remark 1). It is worth noting that the design and analysis of such a monitoring component have been rigorously investigated in [20] and [29]. For completeness, we next provide a brief description of the FDI scheme.

A. Fault Detection and Approximation Scheme

Based on (1), the FDAE is chosen as

$$\dot{\hat{x}}^0 = -\Lambda^0(\hat{x}^0 - x) + \phi(x,u) + \hat{f}(x,u,\hat{\theta}^0) \tag{5}$$

where $\hat{x}^0 \in \Re^n$ is the estimated state vector, $\hat{f} : \Re^n \times \Re^m \times \Re^p \mapsto \Re^n$ is an online approximation model, $\hat{\theta}^0 \in \Re^p$ represents a vector of adjustable weights, and $\Lambda^0 = \mathrm{diag}(\lambda_1^0, \ldots, \lambda_n^0)$, where $-\lambda_i^0 < 0$ is the ith estimator pole. The initial weight vector, $\hat{\theta}^0(0)$, is chosen such that $\hat{f}(x,u,\hat{\theta}^0(0)) = 0, \forall (x,u) \in \mathcal{D}$, which corresponds to the case where a system is in "healthy" (no fault) condition.

A key component of the nonlinear adaptive estimator described by (5) is the *online approximator*, denoted by $\hat{f}$. Each component of $\hat{f}$ has the structure

$$\hat{f}_i(x,u,\hat{\theta}^0) = \sum_{j=1}^{\nu} c_{ij}\varphi_j(x,u,\sigma_j), \qquad c_{ij} \in \Re, \qquad \sigma_j \in \Re^k \tag{6}$$

where $\varphi_j(\cdot,\cdot,\cdot)$ are given parametrized basis functions, and c_{ij} and σ_j are the parameters (weights) to be determined, i.e., $\hat{\theta}^0 \triangleq \mathrm{col}(c_{ij}, \sigma_j : j = 1, \ldots, \nu, i = 1, \ldots, n)$. In the presence of a fault, $\hat{f}$ provides the adaptive structure for approximating online the unknown fault function. This is achieved by adapting the weight vector $\hat{\theta}^0(t)$. The term "on-line approximator" is used to represent nonlinear multivariable approximation models with adjustable parameters [24], [16], [28], such as neural networks, fuzzy logic networks, polynomials, spline functions, etc.

The next step in the construction of the FDAE is the design of the learning algorithm for updating the weights $\hat{\theta}^0$. Let $\epsilon^0(t) \triangleq x(t) - \hat{x}^0(t)$ be the state estimation error. Using techniques from adaptive control [10], [15], the learning algorithm for the online approximator can be chosen as

$$\dot{\hat{\theta}}^0 = \mathcal{P}_{\Theta^0}\{\Gamma^0 Z^\top D[\epsilon^0]\} \tag{7}$$

where the *projection operator* $\mathcal{P}$ restricts the parameter estimation vector $\hat{\theta}^0$ to a predefined compact and convex region $\Theta^0 \subset \Re^p$ to guarantee the stability of the learning algorithm in the presence of network approximation errors, $\Gamma^0 = \Gamma^{0\top} \in \Re^{p\times p}$ is a positive–definite learning rate matrix, and $Z : \Re^n \times \Re^m \times \Re^p \mapsto \Re^{n\times p}$ denotes the gradient matrix of the online approximator with respect to its adjustable weights, i.e., $Z \triangleq \partial\hat{f}(x,u,\hat{\theta}^0)/\partial\hat{\theta}^0$. The *dead-zone* operator $D[\,\cdot\,]$ is defined as

$$D[\epsilon^0(t)] \triangleq \begin{cases} 0, & \text{if } \left|\epsilon_i^0(t)\right| \le \bar{\epsilon}_i^0(t) \qquad \forall i = 1, \ldots, n \\ \epsilon^0(t), & \text{otherwise.} \end{cases}$$

The presence of modeling errors $\eta(x,u,t)$ causes a nonzero state estimation error $\epsilon^0(t)$, even in the absence of a fault. The dead-zone operator $D[\cdot]$ prevents the adaptation of the approximator weights when the modulus of every estimation error component $|\epsilon_i^0(t)|$ does not exceed its own bound $\bar{\epsilon}_i^0(t)$, thereby preventing any false alarms. The time-varying dead-zone bounds $\bar{\epsilon}_i^0(t)$ are chosen as follows, for $0 \le t_0 \le T_0$:

$$\bar{\epsilon}_i^0(t) \triangleq \int_{t_0}^{t} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i(x(\tau),u(\tau),\tau)\,d\tau + \left|\epsilon_i^0(t_0)\right| e^{-\lambda_i^0(t-t_0)}$$

where the first term can be implemented as the output of a linear filter (with the transfer function $1/(s+\lambda_i^0)$ and under zero initial conditions), with the input given by $\bar{\eta}_i(t) = \bar{\eta}_i(x(t),u(t),t)$. The decision scheme for fault detection is as follows.

Fault Detection Decision Scheme: The decision on the occurrence of a fault (detection) is made when the modulus of at least one of the estimation error components $|\epsilon_i^0(t)|$ exceeds its corresponding bound $\bar{\epsilon}_i^0(t)$. The fault detection time T_d is defined as

$$T_d \triangleq \inf \bigcup_{i=1}^{n} \left\{ t > T_0 : |\epsilon_i^0(t)| > \bar{\epsilon}_i^0(t) \right\}. \tag{8}$$

For more details concerning the FDAE (e.g., *fault detectability conditions*, *stability properties* of the learning scheme described by (5) and (7), etc.), we refer the reader to [20].

B. Fault Isolation Scheme

After a fault has been detected, the isolation scheme is activated (see Fig. 2). Each nonlinear adaptive isolation estimator corresponds to one of the possible types of nonlinear faults belonging to the fault class $\mathcal{F}$, that is

$$\begin{aligned} \dot{\hat{x}}^s &= -\Lambda^s(\hat{x}^s - x) + \phi(x,u) + \hat{f}^s(x,u,\hat{\theta}^s) \\ \hat{f}^s(x,u,\hat{\theta}^s) &= \left[\left(\hat{\theta}_1^s\right)^\top g_1^s(x,u), \ldots, \left(\hat{\theta}_n^s\right)^\top g_n^s(x,u) \right]^\top \end{aligned} \tag{9}$$

where $\hat{\theta}_i^s \in \Re^{q_i^s}$, for $i = 1,\ldots,n$, $s = 1,\ldots,N$, is the estimate of the fault parameter vector in the ith state variable. Moreover, $\Lambda^s = \mathrm{diag}(\lambda_1^s,\ldots,\lambda_n^s)$, where $-\lambda_i^s < 0$ are design constants representing the estimator pole locations. For notational simplicity and without loss of generality, in this paper we assume that $\lambda_i^s = \lambda_i$, for all $s = 1,\ldots,N$.

The adaptation in the isolation estimators is due to the unknown parameter vector θ_i^s. The adaptive law for updating each $\hat{\theta}_i^s$ is derived by using the Lyapunov synthesis approach, with the projection operator restricting $\hat{\theta}_i^s$ to the corresponding known set Θ_i^s. Specifically, if we let $\epsilon_i^s(t) \triangleq x_i(t) - \hat{x}_i^s(t)$ be the ith component of the state estimation error vector of the sth estimator, then the following learning algorithm is chosen:

$$\dot{\hat{\theta}}_i^s = \mathcal{P}_{\Theta_i^s}\left\{\Gamma_i^s g_i^s(x,u)\epsilon_i^s\right\} \tag{10}$$

where $\Gamma_i^s > 0$ is a symmetric, positive–definite learning rate matrix. It should be noted that, as the isolation estimators are activated only after the detection of a fault, there is no need to apply the dead-zone operator to the state estimation error.

The fault isolation decision scheme is based on the following intuitive principle: if fault s occurs at some time T_0 and is detected at time T_d, then a set of threshold functions $\mu_i^s(t)$ exist such that the ith component of the state estimation error of the sth estimator satisfies $|\epsilon_i^s(t)| \le \mu_i^s(t), i = 1,\ldots,n$, for all $t \ge T_d$. Consequently, for each $s = 1,\ldots,N$, a set of adaptive threshold functions $\mu_i^s(t)$ can be associated with the sth fault isolation estimator. For a particular s, if $|\epsilon_i^s(t)| > \mu_i^s(t)$ for some $t > T_d$ and some $i = 1,\ldots,n$, then the possibility of the occurrence of fault s can be excluded. Therefore, we use the following decision scheme for fault isolation.

Fault Isolation Decision Scheme: If, for each $r \in \{1,\ldots,N\}\backslash\{s\}$, there exist some time $t^r > T_d$ and some $i \in \{1,\ldots,n\}$ such that $|\epsilon_i^r(t^r)| > \mu_i^r(t^r)$, then the occurrence of fault s is deduced. The fault isolation time is defined as

$$T_{\mathrm{isol}}^s \triangleq \max\{t^r, r \in \{1,\ldots,N\}\backslash\{s\}\}. \tag{11}$$

Clearly, a basic role in the previous fault isolation scheme is played by the adaptive thresholds $\mu_i^s(t)$. Let T_d be the fault detection time given by (8). According to the analysis made in [29], the following threshold functions for fault isolation can be chosen:

$$\begin{aligned} \mu_i^s(t) = \int_{T_d}^{t} e^{-\lambda_i(t-\tau)} &\left[\left(\kappa_i^s(\tau) + e^{-\bar{a}_i(\tau - T_d)}\left|\hat{\theta}_i^s(\tau)\right| \right) \right. \\ &\left. \cdot \left|g_i^s(x(\tau),u(\tau))\right| + \bar{\eta}_i(x(\tau),u(\tau),\tau) \right] d\tau \\ &+ \left|\epsilon_i^s(T_d)\right| e^{-\lambda_i(t-T_d)} \end{aligned} \tag{12}$$

where $\kappa_i^s(t)$ represents the maximum fault-parameter vector estimation error, i.e., $|\theta_i^s - \hat{\theta}_i^s(t)| \le \kappa_i^s(t)$. The form of $\kappa_i^s(t)$ depends on the geometric properties of the compact set Θ_i^s. For instance, assume that the parameter set Θ^s is a hypersphere (or the smallest hypersphere containing the set of all possible $\hat{\theta}_i^s(t)$) with center O_i^s and radius R_i^s; then we have $\kappa_i^s(t) = R_i^s + |\hat{\theta}_i^s(t) - O_i^s|$. It is worth noting that the term $\kappa_i^s(\tau)|g_i^s(x(\tau),u(\tau))|$ is due to the unknown fault parameter vector θ^s (in this paper, we do not assume persistency of excitation). Moreover, $\bar{a}_i$ is assumed to be a known lower bound on the unknown fault-evolution rate a_i, for $i = 1,\ldots,n$. In a sense, $\bar{a}_i$ can be interpreted as a tuning parameter that can be set by exploiting some *a priori* knowledge of the fault developing dynamics. If no specific knowledge of the fault-evolution rate is available, it is always possible to make a cautious (and possibly conservative) choice of a suitably small $\bar{a}_i$.

The adaptive threshold given by (12) can be easily implemented online with linear filtering techniques. Specifically, the first term of (12) can be implemented as the output of a linear filter (with the transfer function $1/(s+\lambda_i)$), with the input $(\kappa_i^s(t) + e^{-\bar{\alpha}_i(t-T_d)}|\hat{\theta}_i^s(t)|)|g_i^s(x(t),u(t))| + \bar{\eta}_i(x(t),u(t),t)$ and under zero initial conditions.

We stress that each term in the threshold function $\mu_i^s(t)$ described by (12) represents a type of uncertainty that enters the fault isolation problem: 1) $\kappa_i^s(\tau)|g_i^s(x(\tau),u(\tau))|$ is due to the parametric uncertainty $\theta_i^s - \hat{\theta}_i^s(t)$; 2) $e^{-\bar{\alpha}_i(\tau-T_d)}|\hat{\theta}_i^s(\tau)||g_i^s(x(\tau),u(\tau))|$ is due to the unknown fault evolution rate α_i; 3) $\bar{\eta}_i$ is due to the modeling uncertainty η_i; and 4) the last term $|\epsilon_i^s(T_d)|e^{-\lambda_i(t-T_d)}$ appears to be due to the initial state estimation error. On the other hand, the capability to isolate the fault depends not only on the thresholds $\mu_i^s(t)$ but also on "how different the faults are from one another." This intuitive, though qualitative, concept is made precise by using the so-called *fault mismatch function* introduced in [29], where the fault isolability condition characterizing the class of faults that can be isolated by the considered isolation scheme is rigorously defined. Some other important properties of the fault isolation scheme, such as stability and *fault isolation time*, have also been investigated in [29].

Remark 4: In the fault-diagnosis literature, there exist several types of observer schemes. For example, within the fault-isolation framework, the *dedicated observer scheme* (DOS) proposed by Clark [3], [5], [6] and the *generalized observer scheme* (GOS) presented by Frank [8] are typically used. The reader is referred to [29] for some further discussions on these observer schemes.

IV. CONTROLLER MODULE

In this section, we describe and analyze the proposed controller module (see Fig. 2). To facilitate the analysis of the feedback control systems, from now on we assume that the general plant given by (1) takes on the following specific structure:

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \phi_i(\bar{x}_i) + \eta_i(x,u,t) + \beta_i(t-T_0) f_i(\bar{x}_i) \\ &\qquad \text{for } 1 \le i \le n-1 \\ \dot{x}_n &= \phi_0(x)u + \phi_n(x) + \eta_n(x,u,t) + \beta_n(t-T_0) f_n(x) \\ y &= x_1 \end{aligned} \tag{13}$$

where $x \triangleq \mathrm{col}(x_1,\ldots,x_n)$ is the state vector, $\bar{x}_i \triangleq \mathrm{col}(x_1,\ldots,x_i), u \in \Re$ is the control input, $y \in \Re$ is the output, ϕ_0 is a nonzero smooth function, and ϕ_i, η_i, and f_i, for $1 \le i \le n$, are generic smooth functions. The control objective is to force the output $y(t)$ to track a given reference signal $y_r(t)$. We assume that $y_r(t)$ and its first n derivatives are known, piecewise continuous and bounded (the jth-order time-derivative of $y_r(t)$ is denoted by $y_r^{(j)}(t)$).

The *known* nominal system dynamics can be expressed as

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \phi_i(\bar{x}_i), \qquad 1 \le i \le n-1 \\ \dot{x}_n &= \phi_0(x)u + \phi_n(x) \\ y &= x_1. \end{aligned}$$

Moreover, the fault functions making up the class $\mathcal{F}$ are assumed to take on the specific form

$$f^s(x) \triangleq \left[(\theta_1^s)^\top g_1^s(x_1), \ldots, (\theta_n^s)^\top g_n^s(x) \right]^\top. \tag{14}$$

The FDAE and FIE estimators are updated as follows:

$$\begin{aligned} \dot{\hat{x}}_i^0 &= -\lambda_i^0 \left(\hat{x}_i^0 - x_i\right) + x_{i+1} + \phi_i(\bar{x}_i) + \hat{f}_i(\bar{x}_i, \hat{\theta}^0), \\ &\qquad \text{for } 1 \le i \le n-1 \\ \dot{\hat{x}}_n^0 &= -\lambda_n^0 \left(\hat{x}_n^0 - x_n\right) + \phi_0(x)u + \phi_n(x) + \hat{f}_n(x, \hat{\theta}^0) \end{aligned} \tag{15}$$

and

$$\begin{aligned} \dot{\hat{x}}_i^s &= -\lambda_i^s \left(\hat{x}_i^s - x_i\right) + x_{i+1} + \phi_i(\bar{x}_i) + \hat{f}_i^s\left(\bar{x}_i, \hat{\theta}_i^s\right), \\ &\qquad \text{for } 1 \le i \le n-1 \\ \dot{\hat{x}}_n^s &= -\lambda_n^s \left(\hat{x}_n^s - x_n\right) + \phi_0(x)u + \phi_n(x) + \hat{f}_n^s\left(x, \hat{\theta}_n^s\right). \end{aligned} \tag{16}$$

In the following, the design and analysis of the fault-tolerant control scheme described in (4) are rigorously investigated for three different operating modes of the closed-loop system: 1) before fault detection, 2) between fault detection and isolation, and 3) after fault isolation.

A. Nominal Controller and the System Stability Before Fault Detection

In this section, we design the nominal controller and investigate the system stability issue before a fault is detected. Using the backstepping methodology [14], a new state vector $z \triangleq \mathrm{col}(z_1,\ldots,z_n)$ is defined recursively by the following coordinate transformation, for $i = 1,\ldots,n$

$$z_i = x_i - \alpha_{i-1}\left(x_1,\ldots,x_{i-1}, y_r,\ldots,y_r^{(i-2)}\right) - y_r^{(i-1)} \tag{17}$$

where

$$\begin{aligned} \alpha_0 &= 0 \\ \alpha_1 &= -c_1 z_1 - c_2 z_1 - \phi_1 \\ \alpha_i &= -c_1 z_i - z_{i-1} - \phi_i - c_2 z_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j}\right)^2 \\ &\quad + \sum_{j=0}^{i-2} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} + \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(x_{j+1} + \phi_j), \\ &\qquad \text{for } i = 2,\ldots,n \end{aligned} \tag{18}$$

where c_1 and c_2 are suitable design constants. Due to the boundedness of η [see (2)], the nominal controller

$$u_0(t) = \frac{1}{\phi_0(x)}\left[\alpha_n + y_r^{(n)}\right] \tag{19}$$

guarantees the stability of the system in the presence of modeling uncertainty for $t < T_0$ (i.e., before the occurrence of any faults); (18) and (19) correspond to the general controller state equations (4) in the case $t < T_d$. In the rest of the paper, for simplicity and without any ambiguity, we shall apply the control laws (4) in three different cases by simply referring to the control variables $u_0(t), u_D(t)$, and $u_I(t)$, respectively.

After a fault has occurred, but before its detection (i.e., for $T_0 \le t < T_d$), on the basis of (13), (15) the ith component of the estimation error, i.e., $\epsilon_i^0 \triangleq x_i - \hat{x}_i^0$, satisfies

$$\dot{\epsilon}_i^0(t) = -\lambda_i^0 \epsilon_i^0(t) + \eta_i(x,u,t) + \beta_i(t-T_0) f_i(\bar{x}_i). \tag{20}$$

As the fault has not been detected yet, the estimation error remains below its dead-zone threshold [see (8)], that is

$$\left|\epsilon_i^0(t)\right| \le \bar{\epsilon}_i^0(t), \qquad \text{for } T_0 \le t < T_d. \tag{21}$$

Clearly, the occurrence of a fault may affect the stabilizing property of the nominal controller (19). To address this issue, we first need the following basic lemma concerning the fault-detection time.

Lemma 1: Suppose that a fault occurs at some time T_0. Moreover, assume that there exist a time interval $[T_1, T_2]$, a scalar $M > 0$, and an index $i \in \{1,\ldots,n\}$ such that for all $t \in [T_1, T_2]$

$$|\beta_i(t-T_0) f_i[\bar{x}_i(t)]| \ge M + 2\bar{\eta}_i[x(t), u(t), t] \tag{22}$$

where $T_1 > T_0, T_2 > T_1 + D(M)$, and $D(M)$ is a time period defined as $D(M) \triangleq \ln(1+4\lambda_i^0\bar{\epsilon}_i^0(T_1)/M)/\lambda_i^0$. Then, an upper bound to the fault detection time is $T_1 + D(M)$. □

Proof: For all $t \in [T_1, T_2]$, from the solution of the error dynamics equation (20) and by using the triangle inequality, we obtain

$$\begin{aligned}\left|\epsilon_i^0(t)\right| \geq & \left|\int_{T_0}^{t} e^{-\lambda_i^0(t-\tau)}\beta_i(\tau-T_0)f_i[\bar{x}_i(\tau)]\,d\tau\right| \\ & - \int_{T_0}^{t} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau \\ & - \left|\epsilon_i^0(T_0)\right| e^{-\lambda_i^0(t-T_0)}. \qquad (23)\end{aligned}$$

The dead-zone threshold for fault detection can be written as

$$\begin{aligned}\bar{\epsilon}_i^0(t) = & \int_{T_0}^{t} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau \\ & + \left|\epsilon_i^0(T_0)\right| e^{-\lambda_i^0(t-T_0)}. \qquad (24)\end{aligned}$$

According to (23) and (24), a sufficient condition for fault detection (i.e., a condition implying that $|\epsilon_i^0(t)| > \bar{\epsilon}_i^0(t)$) is

$$\begin{aligned}\left|\int_{T_0}^{t} e^{-\lambda_i^0(t-\tau)}\beta_i(\tau-T_0)f_i[\bar{x}_i(\tau)]\,d\tau\right| > & \, 2\left|\epsilon_i^0(T_0)\right| e^{-\lambda_i^0(t-T_0)} \\ & + 2\int_{T_0}^{t} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau. \qquad (25)\end{aligned}$$

Recall the definition of T_1 given in Lemma 1. Then, using $\int_{T_0}^{t} = \int_{T_0}^{T_1} + \int_{T_1}^{t}$ and the triangle inequality again, a sufficient condition for (25) to hold is given by

$$\begin{aligned}\left|\int_{T_1}^{t} e^{-\lambda_i^0(t-\tau)}\beta_i(\tau-T_0)f_i[\bar{x}_i(\tau)]\,d\tau\right| > & \, 2\left|\epsilon_i^0(T_0)\right| e^{-\lambda_i^0(t-T_0)} \\ & + \left|\int_{T_0}^{T_1} e^{-\lambda_i^0(t-\tau)}\beta_i(\tau-T_0)f_i[\bar{x}_i(\tau)]\,d\tau\right| \\ & + 2\int_{T_0}^{t} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau. \qquad (26)\end{aligned}$$

Note that $|\epsilon_i^0(T_1)| \leq \bar{\epsilon}_i^0(T_1)$ (if $|\epsilon_i^0(T_1)| > \bar{\epsilon}_i^0(T_1)$, the fault would already have been detected). Hence, from (25), we obtain

$$\begin{aligned}\left|\int_{T_0}^{T_1} e^{-\lambda_i^0(T_1-\tau)}\beta_i(\tau-T_0)f_i[\bar{x}_i(\tau)]\,d\tau\right| & \\ \leq 2\int_{T_0}^{T_1} e^{-\lambda_i^0(T_1-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau & \\ + 2\left|\epsilon_i^0(T_0)\right| e^{-\lambda_i^0(T_1-T_0)}. & \qquad (27)\end{aligned}$$

Furthermore, note that

$$\begin{aligned}& \left|\int_{T_0}^{T_1} e^{-\lambda_i^0(t-\tau)}\beta_i(\tau-T_0)f_i[\bar{x}_i(\tau)]\,d\tau\right| \\ & = e^{-\lambda_i^0(t-T_1)}\left|\int_{T_0}^{T_1} e^{-\lambda_i^0(T_1-\tau)}\beta_i(\tau-T_0)f_i[\bar{x}_i(\tau)]\,d\tau\right|.\end{aligned}$$

Then, if we use (27), condition (26) is guaranteed by the following inequality:

$$\begin{aligned}& \left|\int_{T_1}^{t} e^{-\lambda_i^0(t-\tau)}\beta_i(\tau-T_0)f_i[\bar{x}_i(\tau)]\,d\tau\right| \\ & > 2\int_{T_1}^{t} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau + 4e^{-\lambda_i^0(t-T_0)} \\ & \cdot |\epsilon_i^0(T_0)| + 4\int_{T_0}^{T_1} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau. \qquad (28)\end{aligned}$$

According to (22) and Assumption 1, it follows that the quantity $\beta_i(t-T_0)f_i[\bar{x}_i(t)]$ does not change sign over the time interval $[T_1, T_2]$. Therefore, a sufficient condition for (28) to hold is

$$\begin{aligned}\frac{M}{\lambda_i^0}\left[1-e^{-\lambda_i^0(t-T_1)}\right] > & \, 4\int_{T_0}^{T_1} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau \\ & + 4\left|\epsilon_i^0(T_0)\right| e^{-\lambda_i^0(t-T_0)}.\end{aligned}$$

Note that the left-hand side of the aforementioned inequality is an increasing function of t, whereas the right-hand side is a decreasing function of t. Therefore, the fault-detection time can be obtained by solving the following equation for t:

$$\begin{aligned}\frac{M}{\lambda_i^0}\left(1-e^{-\lambda_i^0(t-T_1)}\right) = & \, 4\int_{T_0}^{T_1} e^{-\lambda_i^0(t-\tau)}\bar{\eta}_i[x(\tau),u(\tau),\tau]\,d\tau \\ & + 4\left|\epsilon_i^0(T_0)\right| e^{-\lambda_i^0(t-T_0)}.\end{aligned}$$

After some algebraic manipulations, we obtain $t = T_1 + \ln[1 + 4\lambda_i^0\bar{\epsilon}_i^0(T_1)/M]/\lambda_i^0$. The proof is completed by letting $D(M) \triangleq t - T_1$. ■

In the presence of a fault but before its detection, some variables in the controlled system may grow unbounded, thus possibly causing also the fault function to become unbounded, that is, the nominal controller (19) may lose its stabilizing properties.

The key issue to be resolved is to ensure that the faulty behavior will be detected before the possible occurrence of an unbounded growth of some state variable. In the following analysis, a contradiction logic will be exploited. More specifically, let us suppose that the fault function $\beta_i(t-T_0)f_i[\bar{x}_i(t)]$ has some *finite escape time* T_e before fault detection, i.e., $\lim_{t\to T_e^-}|\beta_i(t-T_0)f_i[\bar{x}_i(t)]| = \infty$, where $T_0 < T_e < T_d$; more precisely, $\forall N > 0, \exists\delta(N) > 0$, such that

$$|\beta_i(t-T_0)f_i[\bar{x}_i(t)]| > N \qquad \forall t \in (T_e-\delta(N), T_e). \qquad (29)$$

Now, according to Assumption 1, let us denote by $\bar{\eta}_i^+$ a uniform constant bound on the modeling uncertainty, i.e., $\bar{\eta}_i^+ \triangleq \sup_{t\in(T_0,T_d)}\bar{\eta}_i[x(t),u(t),t]$. To analyze stability before fault detection, the following assumption will be used.

Assumption 2: There exists some finite scalar $N > 2\bar{\eta}_i^+$ such that $\delta(N) > D(N-2\bar{\eta}_i^+)$, where $\delta(N)$ is defined in (29) and the function $D(\cdot)$ is defined as in Lemma 1.

In the subsequent analysis, Assumption 2 will be related to the rate of growth of the fault function to infinity. In order to gain a deeper insight into the meaning of Assumption 2, let us first give the following example.

Illustrative Example: Consider the differential equation:

$$\dot{x}(t) = f(x(t)) = x^a(t) \qquad \forall\, a > 0. \tag{30}$$

Without loss of generality, we let $x(0) = 1$ and $a > 1$. Let us interpret $f[x(t)] = x^a(t)$ as the fault function considered in Lemma 1; clearly, $\lim_{t \to 1/(a-1)} f(t) = \infty$. It is easy to show that $\forall N > 0$, we have $\delta(N) = 1/((a-1)N^{(1-1/a)})$. Now, for $N > 2\bar{\eta}_i^+$ and after some algebra, a sufficient condition for Assumption 2 to be satisfied is $N > (4\bar{\epsilon}_i^0(T_1)(a-1))^a$. Hence, if we choose $N > \max\{(4\bar{\epsilon}_i^0(T_1)(a-1))^a, 2\bar{\eta}_i^+\}$, Assumption 2 is satisfied.

The previous example analytically shows how fast a fault function may grow to infinity, while still satisfying Assumption 2. We are now able to prove the following lemma.

Lemma 2: Assume that a fault occurs at time T_0 and that it is detected at some finite time $T_d > T_0$. Then, the fault function $\beta_i(t-T_0)f[\bar{x}_i(t)]$ remains bounded before the fault is detected, i.e., $|\beta_i(t-T_0)f_i[\bar{x}_i(t)]| \leq N$, for some finite positive constant N, for all $t \in (T_0, T_d)$, and for all $1 \leq i \leq n$. □

Proof: Let us suppose that there exists some index $i \in \{1, \ldots, n\}$ such that $\lim_{t \to T_e^-} |\beta_i(t-T_0)f_i[\bar{x}_i(t)]| = \infty$, where $T_e \in (T_0, T_d)$. We choose $N = M + 2\bar{\eta}_i^+$ for some generic $M > 0$. Then, there exists some $\delta(N) > 0$ such that $|\beta_i(t - T_0)f_i[\bar{x}_i(t)]| > N, \forall t \in (T_e - \delta(N), T_e)$. By letting $T_1 \triangleq T_e - \delta(N)$ and by using Lemma 1 and Assumption 2, we obtain an upper bound to the fault detection time; it is expressed as $T_1 + D(N - 2\bar{\eta}_i^+) = T_e - \delta(N) + D(N - 2\bar{\eta}_i^+) < T_e$, thus proving that the fault is detected prior to the finite escape time. ■

We can now analyze the stability properties of the controlled system before the detection of a fault.

Theorem 1 (Stability Before Fault Detection): Suppose that a fault occurs at time T_0 and consider the time window $[T_0, T_d)$. Then, the nominal controller given by (19) has the following properties.

1) The system state variables remain bounded, i.e., $x(t)$ is uniformly bounded for all $t \in (T_0, T_d)$.
2) There exist positive constants k_1, k_2 and a continuous function $\bar{\nu}(t)$ (depending on the modeling uncertainty η_i and the fault function $\beta_i f_i$) such that, for all $t_f \in (T_0, T_d)$, the new state variable $z(t)$ defined in (17) satisfies

$$\int_{T_0}^{t_f} z(t)^\top z(t)\, dt \leq k_1 + k_2 \int_{T_0}^{t_f} \bar{\nu}(t)^\top \bar{\nu}(t)\, dt. \qquad \square$$

Proof: According to the system model (13) and the coordinate transformation described by (17) and (18), the time-derivative of the new state variable z_1 is

$$\begin{aligned}\dot{z}_1 &= x_2 + \phi_1(y) + \eta_1(x,u,t) + \beta_1(t-T_0)f_1(y) - \dot{y}_r \\ &= z_2 + \alpha_1(y, y_r) + \phi_1(y) + \eta_1 + \beta_1(t-T_0)f_1(y) \\ &= z_2 - c_1 z_1 - c_2 z_1 + \eta_1 + \beta_1(t-T_0)f_1(y)\end{aligned} \tag{31}$$

Similarly, for $i = 2, \ldots, n-1$, the time-derivative of z_i can be recursively obtained as follows:

$$\begin{aligned}\dot{z}_i &= \dot{x}_i - y_r^{(i)} - \sum_{j=0}^{i-2} \frac{\partial \alpha_{i-1}}{\partial y_r^{(j)}} y_r^{(j+1)} - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j} \dot{x}_j \\ &= x_{i+1} - \alpha_i + \eta_i + \beta_i f_i - y_r^{(i)} - c_1 z_i - z_{i-1} \\ &\quad - c_2 z_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j}\right)^2 - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(\eta_j + \beta_j f_j) \\ &= z_{i+1} - c_1 z_i - z_{i-1} - c_2 z_i \sum_{j=1}^{i-1} \left(\frac{\partial \alpha_{i-1}}{\partial x_j}\right)^2 \\ &\quad - \sum_{j=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_j}(\eta_j + \beta_j f_j) + \eta_i + \beta_i f_i.\end{aligned} \tag{32}$$

Likewise, for $i = n$, we have

$$\begin{aligned}\dot{z}_n &= \phi_0 u - \alpha_n + \eta_n + \beta_n f_n - y_r^{(n)} - c_1 z_n - z_{n-1} \\ &\quad - c_2 z_n \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j}\right)^2 - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}(\eta_j + \beta_j f_j).\end{aligned}$$

By substituting the nominal control law (19) into the aforementioned equation, we obtain

$$\begin{aligned}\dot{z}_n &= -c_1 z_n - z_{n-1} - c_2 z_n \sum_{j=1}^{n-1} \left(\frac{\partial \alpha_{n-1}}{\partial x_j}\right)^2 + \eta_n + \beta_n f_n \\ &\quad - \sum_{j=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_j}(\eta_j + \beta_j f_j).\end{aligned}$$

Let $\gamma \triangleq \mathrm{col}(\gamma_1, \gamma_{ij}, i = 2, \ldots, n, j = 1, \ldots, i-1)$, where $\gamma_1 \triangleq -1$ and $\gamma_{ij} \triangleq \partial \alpha_{i-1}/\partial x_j$. Then, according to the previous equations, the system dynamics in the new coordinates given by (17) can be rewritten as

$$\dot{z} = \bar{A}z - h(\gamma, z) - \omega(\gamma, t) - \nu(t) \tag{33}$$

where $\bar{A}$ can be defined in a straightforward way, the components of the vector field $h \triangleq \mathrm{col}(h_1, \ldots, h_n)$ are defined by $h_1 = c_2 z_1 \gamma_1^2$ and $h_i(\gamma, z) = c_2 z_i \sum_{j=1}^{i-1} \gamma_{ij}^2$, for $2 \leq i \leq n$, and the components of the vector field $\omega \triangleq \mathrm{col}(\omega_1, \ldots, \omega_n)$ are defined by $\omega_1 = \gamma_1(\eta_1 + \beta_1 f_1), \omega_i = \sum_{j=1}^{i-1} \gamma_{ij}(\eta_i + \beta_i f_i)$, for $2 \leq i \leq n$, and $\nu(t) \triangleq \mathrm{col}(0, -\eta_2 - \beta_2 f_2, \ldots, -\eta_n - \beta_n f_n)$.

Let us now consider the Lyapunov function candidate $V = (1/2)z^\top z$. The time derivative of V along the solution of (33) is given by $\dot{V} = z^\top[\bar{A}z - h(\gamma, z) - \omega(\gamma, t) - \nu(t)]$. Noting that $z^\top \bar{A} z = -\sum_{i=1}^n c_1 z_i^2$, we have

$$\begin{aligned}\dot{V} &= -\frac{c_1}{2} z^\top z - c_2 \left[z_1^2 \gamma_1^2 + \frac{1}{c_2} z_1 \gamma_1(\eta_1 + \beta_1 f_1)\right] \\ &\quad - c_2 \sum_{i=2}^{n} \sum_{j=1}^{i-1} \left[z_i^2 \gamma_{ij}^2 + \frac{1}{c_2}\gamma_{ij}(\eta_i + \beta_i f_i) z_i\right] \\ &\quad - \sum_{i=1}^{n} \left[\frac{c_1}{2} z_i^2 + z_i \nu_i(t)\right].\end{aligned}$$

Furthermore, from completing the squares, it follows that

$$\begin{aligned}\dot{V} &\leq -\frac{c_1}{2}z^\top z + \frac{n}{4c_2}\sum_{i=1}^{n}(\eta_i + \beta_i f_i)^2 + \frac{1}{2c_1}\sum_{i=1}^{n}\nu_i^2 \\ &\leq -\frac{c_1}{2}z^\top z + \frac{1}{2c_3}\bar{\nu}(t)^\top \bar{\nu}(t) \end{aligned} \tag{34}$$

where each component of $\bar{\nu}(t)$ is given by $\bar{\nu}_i(t) \triangleq [(\eta_i + \beta_i f_i)^2 + \nu_i^2]^{1/2}$, and $c_3 \triangleq \min(c_1, 2c_2/n)$. Inequality (34) guarantees that $\dot{V} \leq 0$ if $z^\top z \geq 1/(c_3 c_1)\bar{\nu}^\top \bar{\nu}$. From Lemma 2 and Assumption 1, i.e., the boundedness of the uncertainty η_i and the fault function $\beta_i f_i$, we deduce the boundedness of $\bar{\nu}(t)$ and hence the boundedness of $z(t)$. According to (17), the boundedness of $z(t)$ and y_r and its derivatives imply that $x(t)$ is bounded. Finally, by integrating (34) from $t = T_0$ to $t = t_f$, we obtain

$$\int_{T_0}^{t_f} z(t)^\top z(t)\,dt \leq \frac{c_1}{2}[V(T_0) - V(t_f)] + \frac{1}{c_3 c_1}\int_{T_0}^{t_f} \bar{\nu}(t)^\top \bar{\nu}(t)\,dt.$$

By defining $k_1 \triangleq \sup_{t_f \geq T_0} \frac{c_1}{2}[V(T_0) - V(t_f)]$ and $k_2 \triangleq 1/(c_3 c_1)$, the proof is concluded. ■

It is worth noting that the boundedness analysis reported in Theorem 1 is useful to ensure that the nominal controller is able to guarantee a "reasonable" performance after the fault occurrence and before any information provided by the monitoring module is available (i.e., for $T_0 \leq t < T_d$). Clearly, this performance may degrade quite rapidly, depending on the specific fault that has occurred. Hence, an early detection decision (i.e., a small value of $T_d - T_0$) would be very beneficial to reconfigure the controller; this will be the subject of the next section.

B. First Controller Reconfiguration: Accommodation Before Fault Isolation

After the fault is detected at time $t = T_d$, the isolation estimators described in Section III-B are activated to determine the particular type of fault that has occurred. Starting from T_d, as (21) is no longer satisfied, the fault function $\beta_i f_i$ may grow unbounded. Therefore, the nominal controller $u_0(t)$ has to be reconfigured to ensure the system stability and some tracking performances after fault detection. In the following, we describe the design of the fault-tolerant controller $u_D(t)$ defined in (4), using stable adaptive tracking techniques [18], [13], [14].

Before the fault is isolated, no information about the fault function is available. Online approximators such as neural-network models can be used to estimate the unknown fault function $\beta_i f_i$. Specifically, with reference to the general structure given by (6), we consider linearly parametrized networks (e.g., radial-basis-function networks with fixed centers and variances) described as

$$\hat{f}_i(\bar{x}_i, \hat{\theta}_i) = \sum_{j=1}^{p_i} c_{ij}\varphi_{ij}(\bar{x}_i) \tag{35}$$

where $\hat{\theta}_i \in \Re^{p_i}, \hat{\theta}_i \triangleq \mathrm{col}(c_{ij}, j = 1, \ldots, p_i)$ denote the adjustable weights of the online linear approximation model, and $\varphi_{ij}(\bar{x}_i)$ represent the fixed network basis functions. Therefore, the system model (13) can be rewritten as

$$\begin{aligned}\dot{x}_i &= x_{i+1} + \phi_i(\bar{x}_i) + \hat{f}_i(\bar{x}_i, \theta_i^*) + \beta_i\delta_i(\bar{x}_i) + \eta_i \\ &\quad + (\beta_i - 1)\hat{f}_i(\bar{x}_i, \theta_i^*), \qquad \text{for } i = 1, \ldots, n-1 \\ \dot{x}_n &= \phi_0(x)u + \phi_n(x) + \hat{f}_n(x, \theta_n^*) + \beta_n\delta_n(x) + \eta_n \\ &\quad + (\beta_n - 1)\hat{f}_n(x, \theta_n^*) \end{aligned} \tag{36}$$

where $\delta_i(\bar{x}_i) \triangleq f_i(\bar{x}_i) - \hat{f}_i(\bar{x}_i, \theta_i^*)$ is the network approximation error for the ith network, and θ_i^* is the optimal weight vector given by

$$\theta_i^* \triangleq \arg \inf_{\hat{\theta}_i \in \Re^{p_i}} \left\{ \sup_{\bar{x}_i \in \bar{\mathcal{X}}_i} |f_i(\bar{x}_i) - \hat{f}_i(\bar{x}_i, \hat{\theta}_i)| \right\}$$

where $\bar{\mathcal{X}}_i \subseteq \Re^i$ denotes the set to which the variables $\bar{x}_i$ belong for all possible modes of behavior of the controlled system. To simplify the subsequent analysis, in the following we assume that the bounding conditions are global, so we set $\bar{\mathcal{X}}_i = \Re^i$, and we consider the global tracking problem. For each network, we make the following assumption on the network approximation error:

Assumption 3: For each $i = 1, \ldots, n$

$$|\delta_i(\bar{x}_i)| \leq \psi_{i\delta} s_{i\delta}(\bar{x}_i) \tag{37}$$

where $\psi_{i\delta} \geq 0$ are unknown bounding parameters and $s_{i\delta} : \Re^i \mapsto \Re^+$ are known smooth bounding functions.

The system described by (36) and (37) is characterized by two types of uncertainty: 1) parametric uncertainty, which arises due to the unknown network weights θ_i^*; and 2) bounding uncertainty, which arises due to the unknown bounding parameters $\psi_{i\delta}$ and the unknown incipient fault time profile β_i. We let $\tilde{\theta}_i(t) \triangleq \hat{\theta}_i(t) - \theta_i^*$ denote the network weight estimation error, and $\tilde{\psi}(t) \triangleq \hat{\psi}(t) - \psi_m$ represent the corresponding bounding parameter error, where ψ_m is an unknown constant expressed as

$$\psi_m \triangleq \sup_{\substack{1 \leq i \leq n \\ t \geq T_d}} \max\{\beta_i(t - T_0)\psi_{i\delta}, |[\beta_i(t - T_0) - 1]\theta_i^*|\} \tag{38}$$

where T_d is the fault detection time defined in (8). Note that the fault time profile $\beta_i(t - T_0)$ satisfies $0 \leq \beta_i \leq 1$. Then, the finite constant ψ_m defined by (38) always exists. For the sake of compactness of notation, we let

$$\bar{\theta}_i \triangleq \mathrm{col}(\hat{\theta}_1, \ldots, \hat{\theta}_i) \quad \bar{y}_r^{(i)} \triangleq \mathrm{col}\Big(y_r, y_r^{(1)}, \ldots, y_r^{(i)}\Big).$$

We now proceed to present the two-step fault-tolerant control design.

Step 1) Backstepping design procedure

We first rewrite the linear approximator (35) as

$$\hat{f}_i(\bar{x}_i, \hat{\theta}_i) = (\hat{\theta}_i)^\top \varphi_i(\bar{x}_i)$$

where $\varphi_i(\bar{x}_i) \triangleq \mathrm{col}(\varphi_{ij}(\bar{x}_i), j = 1, \ldots, p_i)$. Consider a new state vector $z \triangleq \mathrm{col}(z_1, \ldots, z_n)$ defined by the following change of coordinates:[1]

$$z_i = x_i - \alpha_{i-1} - y_r^{(i-1)}, \qquad \text{for } i = 1, \ldots, n \tag{39}$$

where the intermediate control functions are given by

$$\begin{aligned}
\alpha_0 &= 0\\
\alpha_1 &= -c_1 z_1 - \phi_1 - (\hat{\theta}_1)^\top \varphi_1 + \rho_1(y, \hat{\theta}_1, \hat{\psi}, y_r)\\
\alpha_i &= -z_{i-1} - c_i z_i - \phi_i - (\hat{\theta}_i)^\top \varphi_i(\bar{x}_i)\\
&\quad + \sum_{k=1}^{i-1} \left\{ \frac{\partial \alpha_{i-1}}{\partial x_k} [x_{k+1} + \phi_k + (\hat{\theta}_k)^\top \varphi_k(\bar{x}_k)] \right\}\\
&\quad + \sum_{k=1}^{i-1} \left\{ \frac{\partial \alpha_{i-1}}{\partial y_r^{(k-1)}} y_r^{(k)} + \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_k} \tau_{ki} \right\}\\
&\quad - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} (\varphi_k(\bar{x}_k))^\top \Gamma_k \sum_{l=k}^{i-2} \left(\frac{\partial \alpha_l}{\partial \hat{\theta}_k} \right)^\top z_{l+1}\\
&\quad + \rho_i[\bar{x}_i, \bar{\theta}_i, \hat{\psi}, \bar{y}_r^{(i-1)}], \qquad \text{for } i = 2, \ldots, n.
\end{aligned}$$

In these equations, ρ_i denotes a smooth function to be defined later on by the adaptive bounding control design procedure, and the intermediate adaptive functions τ_{ki} are recursively updated as follows:

$$\tau_{11} \triangleq \Gamma_1 \left[\varphi_1(x_1) z_1 - \sigma \left(\hat{\theta}_1 - \theta_1^0 \right) \right] \tag{40}$$

where $\sigma > 0$ is a design constant, and for $i = 2, \ldots, n$

$$\begin{aligned}
\tau_{ki} &\triangleq \tau_{k(i-1)} - \Gamma_k z_i \frac{\partial \alpha_{i-1}}{\partial x_k} \varphi_k(\bar{x}_k), \qquad 1 \le k \le i-1\\
\tau_{ii} &\triangleq \Gamma_i \left[\varphi_i(\bar{x}_k) z_i - \sigma \left(\hat{\theta}_i - \theta_i^0 \right) \right].
\end{aligned} \tag{41}$$

By using (36) and (39), the derivative of the new state variable z_1 can now be expressed as follows:

$$\dot{z}_1 = z_2 + \alpha_1 + \phi_1 + \hat{f}_1(x_1, \theta_1^*) + \beta_1 \delta_1 + (\beta_1 - 1) \hat{f}_1(x_1, \theta_1^*) + \eta_1.$$

Consider the intermediate Lyapunov function $V_1 = (1/2) z_1^2 + (1/2) \tilde{\theta}_1^\top \Gamma_1^{-1} \tilde{\theta}_1 + (1/2\gamma_\psi) \tilde{\psi}^2$, where γ_ψ is another design constant. The time derivative of V_1 is given by

$$\dot{V}_1 = -c_1 z_1^2 - \sigma \tilde{\theta}_1^\top \left(\hat{\theta}_1 - \theta_1^0 \right) + z_1 z_2 + \tilde{\theta}_1^\top \Gamma_1^{-1} (\dot{\hat{\theta}}_1 - \tau_{11}) + \Lambda_1$$

τ_{11} being defined in (40), and

$$\Lambda_1 \triangleq z_1 (\rho_1 + \beta_1 \delta_1 + (\beta_1 - 1) \hat{f}_1(x_1, \theta_1^*) + \eta_1) + \gamma_\psi^{-1} \tilde{\psi} \dot{\hat{\psi}}. \tag{42}$$

Similarly, for $i = 2, \ldots, n-1$, the derivatives of z_i can be recursively obtained as

$$\begin{aligned}
\dot{z}_i &= z_{i+1} + \alpha_i + \phi_i + \hat{f}_i(\bar{x}_i, \theta_i^*) + \beta_i \delta_i\\
&\quad + (\beta_i - 1) \hat{f}_i(\bar{x}_i, \theta_i^*) + \eta_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\psi}} \dot{\hat{\psi}} - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial \hat{\theta}_k} \dot{\hat{\theta}}_k\\
&\quad - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} [x_{k+1} + \phi_k + \hat{f}_k(\bar{x}_k, \theta_k^*) + \beta_k \delta_k + \eta_k]\\
&\quad - \sum_{k=1}^{i-1} \left[\frac{\partial \alpha_{i-1}}{\partial x_k} (\beta_k - 1) \hat{f}_k(\bar{x}_k, \theta_k^*) + \frac{\partial \alpha_{i-1}}{\partial y_r^{(k-1)}} y_r^{(k)} \right].
\end{aligned}$$

[1]For the sake of notational simplicity and without any risk of ambiguity, we do not use different symbols for the new state variables z and the related quantities.

Let $V_i = V_{i-1} + (1/2) z_i^2 + (1/2) \tilde{\theta}_i^\top \Gamma_i^{-1} \tilde{\theta}_i$. Using (40) and (41), after some algebraic manipulations, it can be shown that the time derivative of V_i is described by

$$\begin{aligned}
\dot{V}_i &= -\sum_{k=1}^{i} \left(c_k z_k^2 + \sigma \tilde{\theta}_k^\top \left(\hat{\theta}_k - \theta_k^0 \right) \right) + z_i z_{i+1} + \Lambda_i\\
&\quad + \sum_{k=1}^{i} \left[\left(\tilde{\theta}_k^\top \Gamma_k^{-1} - \sum_{l=k}^{i-1} z_{l+1} \frac{\partial \alpha_l}{\partial \hat{\theta}_k} \right) (\dot{\hat{\theta}}_k - \tau_{ki}) \right]
\end{aligned}$$

where

$$\begin{aligned}
\Lambda_i &\triangleq \Lambda_{i-1} + z_i \Bigg\{ \rho_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\psi}} \dot{\hat{\psi}} + (\beta_i - 1) \hat{f}_i(\bar{x}_i, \theta_i^*)\\
&\quad - \sum_{k=1}^{i-1} \frac{\partial \alpha_{i-1}}{\partial x_k} [\beta_k \delta_k + (\beta_k - 1) \hat{f}_k(\bar{x}_k, \theta_k^*) + \eta_k]\\
&\quad + \beta_i \delta_i + \eta_i \Bigg\}.
\end{aligned} \tag{43}$$

In the final step, we consider the overall Lyapunov function candidate

$$V = V_{n-1} + \frac{1}{2} z_n^2 + \frac{1}{2} \tilde{\theta}_n^\top \Gamma_n^{-1} \tilde{\theta}_n. \tag{44}$$

On the basis of (39)–(41), it can be shown that the time derivative of V is given by

$$\begin{aligned}
\dot{V} &= \sum_{k=1}^{n} \left(\tilde{\theta}_k^\top \Gamma_k^{-1} - \sum_{l=k}^{n-1} z_{l+1} \frac{\partial \alpha_l}{\partial \hat{\theta}_k} \right) (\dot{\hat{\theta}}_k - \tau_{kn})\\
&\quad + z_n \left[\phi_0 u - \alpha_n - y_r^{(n)} \right] + \Lambda_n \left[\bar{x}_n, \bar{\theta}_n, \hat{\psi}, \bar{y}_r^{(n-1)} \right]\\
&\quad - \sum_{k=1}^{n} \left[c_k z_k^2 + \sigma \tilde{\theta}_k^\top (\hat{\theta}_k - \theta_k^0) \right]
\end{aligned}$$

where

$$\begin{aligned}
\Lambda_n &\triangleq \Lambda_{n-1} + z_n \Bigg\{ \rho_n - \frac{\partial \alpha_{n-1}}{\partial \hat{\psi}} \dot{\hat{\psi}} + (\beta_n - 1) \hat{f}_n(\bar{x}_n, \theta_n^*)\\
&\quad - \sum_{k=1}^{n-1} \frac{\partial \alpha_{n-1}}{\partial x_k} [\beta_k \delta_k + (\beta_k - 1) \hat{f}_k(\bar{x}_k, \theta_k^*) + \eta_k]\\
&\quad + \beta_n \delta_n + \eta_n \Bigg\}.
\end{aligned}$$

We choose the fault-tolerant controller $u_D(t)$ defined in (4) and adaptive laws for updating $\hat{\theta}_k(t)$ as follows:

$$u_D(t) = \left[\alpha_n + y_r^{(n)} \right] \Big/ \phi_0 \tag{45}$$

$$\dot{\hat{\theta}}_k(t) = \tau_{kn}, \qquad 1 \le k \le n. \tag{46}$$

This operation yields

$$\dot{V} = -\sum_{k=1}^{n} \left[c_k z_k^2 + \sigma \tilde{\theta}_k^\top \left(\hat{\theta}_k - \theta_k^0 \right) \right] + \Lambda_n. \tag{47}$$

Step 2) Adaptive bounding design procedure

We now consider the bounding uncertainty Λ_n, the recursive design of the bounding control function ρ_i, for $1 \le i \le n$, and the adaptive law for the bounding estimate $\hat{\psi}(t)$. Let us first observe that, for any $\epsilon > 0$ and for any $q \in \Re$, the hyperbolic tangent function fulfills

$$0 \leq |q| - q \tanh\left(\frac{q}{\epsilon}\right) \leq k\epsilon \tag{48}$$

where k is a constant that satisfies $k = e^{-(k+1)}$ (i.e., $k \simeq 0.2785$). By using (42) and Assumption 3, we have

$$\begin{aligned}\Lambda_1 &\leq |z_1|[|\beta_1\psi_{1\delta}|s_{1\delta}(x_1) + |(\beta_1 - 1)\theta_1^*||\varphi_1(x_1)| + |\eta_1|] \\ &\quad + z_1\rho_1 + \gamma_\psi^{-1}\tilde{\psi}\dot{\hat{\psi}} \\ &\leq z_1\rho_1 + \gamma_\psi^{-1}\tilde{\psi}\dot{\hat{\psi}} + |z_1|\psi_m s_1(x_1) + |z_1|\bar{\eta}_1(x, u, t)\end{aligned}$$

where $s_1(x_1) \triangleq s_{1\delta}(x_1) + |\varphi_1(x_1)|$. By choosing the bounding control function ρ_1 and the intermediate adaptation function ν_1 as $\rho_1 \triangleq -\hat{\psi}w_1 - \zeta_1\tanh(z_1\zeta_1/\epsilon), \nu_1 \triangleq \gamma_\psi[z_1w_1 - \sigma(\hat{\psi} - \psi^0)]$, $w_1 \triangleq s_1\tanh(z_1s_1/\epsilon), \zeta_1 \triangleq \bar{\eta}_1$, and by using (48), we obtain

$$\begin{aligned}\Lambda_1 &\leq \gamma_\psi^{-1}\tilde{\psi}(\dot{\hat{\psi}} - \gamma_\psi z_1w_1) + \psi_m(|z_1s_1| - z_1w_1) + |z_1\zeta_1| \\ &\quad - z_1\zeta_1\tanh\left(\frac{z_1\zeta_1}{\epsilon}\right) \\ &\leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + k\epsilon\psi_m + k\epsilon + \gamma_\psi^{-1}\tilde{\psi}(\dot{\hat{\psi}} - \nu_1). \quad (49)\end{aligned}$$

In the aforementioned definition of ν_1, the constant $\psi^0 \geq 0$ is a positive design constant that can be used to enhance the tracking performance in the case where some *a priori* estimate of the unknown ψ_m is available.

For $i = 2$, by (43) and (49), we have

$$\begin{aligned}\Lambda_2 &= \Lambda_1 + z_2\left\{\beta_2\delta_2 + (\beta_2 - 1)\hat{f}_2(\bar{x}_2, \theta_2^*) + \eta_2 \right.\\ &\quad \left. - \frac{\partial\alpha_1}{\partial x_1}[\beta_1\delta_1 + (\beta_1 - 1)\hat{f}_1(x_1, \theta_1^*) + \eta_1]\right\} \\ &\quad + z_2\rho_2 - z_2\frac{\partial\alpha_1}{\partial\hat{\psi}}\dot{\hat{\psi}} \\ &\leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + k\epsilon\psi_m + k\epsilon + \gamma_\psi^{-1}\tilde{\psi}(\dot{\hat{\psi}} - \nu_1) \\ &\quad + z_2\rho_2 - z_2\frac{\partial\alpha_1}{\partial\hat{\psi}}\dot{\hat{\psi}} + \psi_m|z_2|s_2 + |z_2|\left(\bar{\eta}_2 + \left|\frac{\partial\alpha_1}{\partial x_1}\right|\bar{\eta}_1\right)\end{aligned}$$

where $s_2 \triangleq s_{2\delta} + |\varphi_2(\bar{x}_2)| + |\partial\alpha_1/\partial x_1|[s_{1\delta}(x_1) + |\varphi_1(x_1)|]$. By choosing the bounding control function ρ_2 and the intermediate adaptation function ν_2 as $\rho_2 \triangleq -\hat{\psi}w_2 + (\partial\alpha_1)/(\partial\hat{\psi})\nu_2 - \zeta_2\tanh(z_2\zeta_2/\epsilon), \nu_2 \triangleq \nu_1 + \gamma_\psi z_2w_2$, with $w_2 \triangleq s_2\tanh((z_2s_2)/\epsilon)$ and $\zeta_2 \triangleq \bar{\eta}_2 + |(\partial\alpha_1)/(\partial x_1)|\bar{\eta}_1$, we obtain

$$\begin{aligned}\Lambda_2 &\leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + 2k\epsilon\psi_m + 2k\epsilon + (\dot{\hat{\psi}} - \nu_2) \\ &\quad \cdot\left(\gamma_\psi^{-1}\tilde{\psi} - z_2\frac{\partial\alpha_1}{\partial\hat{\psi}}\right).\end{aligned}$$

For $3 \leq i \leq n$, from (43) and via mathematical induction, we have

$$\begin{aligned}\Lambda_i &\leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + (i-1)k\epsilon\psi_m + (i-1)k\epsilon + z_i\rho_i \\ &\quad + \left(\gamma_\psi^{-1}\tilde{\psi} - \sum_{k=1}^{i-2} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\psi}}\right)(\dot{\hat{\psi}} - \nu_{i-1}) + \psi_m|z_i|s_i \\ &\quad - z_i\frac{\partial\alpha_{i-1}}{\partial\hat{\psi}}\dot{\hat{\psi}} + |z_i|\left(\bar{\eta}_i + \sum_{k=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_k}\right|\bar{\eta}_k\right)\end{aligned} \tag{50}$$

where $s_i \triangleq s_{i\delta} + |\varphi_i(\bar{x}_i)| + \sum_{k=1}^{i-1}|(\partial\alpha_{i-1})/(\partial x_k)|[s_{k\delta} + |\varphi_k(\bar{x}_k)|]$.

The bounding control function ρ_i and the intermediate adaptation function ν_i are chosen to be

$$\begin{aligned}\rho_i &\triangleq -\hat{\psi}w_i + \frac{\partial\alpha_{i-1}}{\partial\hat{\psi}}\nu_i + \gamma_\psi w_i\sum_{k=1}^{i-2} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\psi}} \\ &\quad - \zeta_i\tanh\left(\frac{z_i\zeta_i}{\epsilon}\right)\end{aligned} \tag{51}$$

$$\nu_i \triangleq \nu_{i-1} + \gamma_\psi z_iw_i \quad w_i \triangleq s_i\tanh\left(\frac{z_is_i}{\epsilon}\right) \tag{52}$$

$$\zeta_i \triangleq \bar{\eta}_i + \sum_{k=1}^{i-1}\left|\frac{\partial\alpha_{i-1}}{\partial x_k}\right|\bar{\eta}_k. \tag{53}$$

By substituting (51)–(53) into (50), and after some algebraic manipulations, for $i = n$, we finally obtain

$$\begin{aligned}\Lambda_n &\leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + \left(\gamma_\psi^{-1}\tilde{\psi} - \sum_{k=1}^{n-1} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\psi}}\right)(\dot{\hat{\psi}} - \nu_n) \\ &\quad + nk\epsilon\psi_m + nk\epsilon.\end{aligned}$$

Therefore, by choosing the adaptive law

$$\dot{\hat{\psi}} = \nu_n = \Gamma_\psi\left[\sum_{k=1}^{n} z_kw_k - \sigma(\hat{\psi} - \psi^0)\right] \tag{54}$$

we have

$$\Lambda_n \leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + nk\epsilon\psi_m + nk\epsilon. \tag{55}$$

By substituting (55) into (47), and by completing the square for each parameter estimate, the following inequality is obtained:

$$\begin{aligned}\dot{V} &\leq -\sum_{k=1}^{n} c_kz_k^2 - \frac{\sigma}{2}\left(\sum_{k=1}^{n}|\tilde{\theta}_k|^2 + |\tilde{\psi}|^2\right) + nk\epsilon\psi_m \\ &\quad + nk\epsilon + \frac{\sigma}{2}\left(\sum_{k=1}^{n}\left|\theta_k^* - \theta_k^0\right|^2 + |\psi_m - \psi^0|^2\right).\end{aligned}$$

Therefore, the Lyapunov function V satisfies

$$\dot{V} \leq -cV + b \tag{56}$$

where $c \triangleq \min_{1 \leq i \leq n}\min\{2c_i, \sigma/(\lambda_{\min}(\Gamma_i^{-1})), \sigma\gamma_\psi\}$ and $b \triangleq nk\epsilon\psi_m + nk\epsilon + (\sigma/2)(\sum_{k=1}^{n}|\theta_k^* - \theta_k^0|^2 + |\psi_m - \psi^0|^2)$. Now, if we let $\bar{\kappa} \triangleq b/c > 0$, by (56) we have

$$0 \leq V(t) \leq \bar{\kappa} + [V(0) - \bar{\kappa}]e^{-ct}. \tag{57}$$

Therefore, $z(t), \hat{\theta}(t), \hat{\psi}(t)$, and $x(t)$ are uniformly bounded. Furthermore, by using (44) and (57), we obtain that, given any $\bar{\epsilon} > \sqrt{2\bar{\kappa}}$, there exists some time T such that, for all $t \geq T$, the output $y = x_1$ satisfies $|y(t) - y_r(t)| \leq \bar{\epsilon}$.

The aforementioned design and analysis procedure is summarized in the following important theorem.

Theorem 2: Suppose that the bounding Assumption 3 holds globally. Then, if a fault is detected, the adaptive fault-tolerant control law (45), the weight parameter adaptive law (46) and the bounding parameter adaptive law (54) guarantee that

1) all the signal and parameter estimates are uniformly bounded, i.e., $z(t), \hat{\theta}(t), \hat{\psi}(t)$, and $x(t)$ are bounded for all $t \in (T_0, T_d)$;

2) given any $\bar{\epsilon} > (2b/c)^{1/2}$, there exists $T(\bar{\epsilon})$ such that $|y(t) - y_r(t)| \leq \bar{\epsilon}$, for all $t > T(\bar{\epsilon})$. □

Theorem 2 guarantees the system's stability and tracking performances after the fault has been detected. As no further information about the fault is available at this stage, Assumption 3 provides a bounding function on the network approximation error for the design of the fault-tolerant control law (45). However, this critical assumption may result in conservative bounds in (37); this justifies the fault-isolation and controller-reconfiguration procedure analyzed in the next section.

C. Second Controller Reconfiguration: Accommodation After Fault Isolation

Let us now assume that the isolation procedure described in Section III-B provides the information that fault s has been isolated at time T_{isol}^s, as defined by (11). Then

$$\begin{aligned} \dot{x}_i &= x_{i+1} + \phi_i(\bar{x}_i) + \eta_i + (\theta_i^s)^\top g_i^s(\bar{x}_i) \\ &\quad + (\beta_i(t - T_0) - 1)(\theta_i^s)^\top g_i^s(\bar{x}_i), \qquad i = 1, \ldots, n-1 \\ \dot{x}_n &= \phi_0(x)u + \phi_n(x) + \eta_n + (\theta_n^s)^\top g_n^s(x) \\ &\quad + (\beta_n(t - T_0) - 1)(\theta_n^s)^\top g_n^s(x) \\ y &= x_1. \end{aligned} \tag{58}$$

If we compare (58) with the system model (36) before fault isolation, we can see that the network approximation error $\delta_i(\bar{x}_i)$ no longer exists. Consequently, the critical Assumption 3 can be removed for the design of the fault-tolerant controller $u_I^s(t)$ [see (4) again] used after fault isolation. We let $\tilde{\theta}_i^s \triangleq \hat{\theta}_i^s(t) - \theta_i^s$ denote the fault parameter vector estimation error, and $\tilde{\psi}(t) \triangleq \hat{\psi}(t) - \psi_m$ represent the corresponding bounding parameter error, where ψ_m is an unknown constant defined as $\psi_m \triangleq \sup_{1 \leq i \leq n, t \geq T_{\text{isol}}^s} \max\{|[\beta_i(t - T_0) - 1]\theta_i^s|\}$. The remaining procedures for designing the stable adaptive control $u_I^s(t)$ are analogous to that for $u_D(t)$ described in Section IV-B; thus, for the sake of brevity, several details are omitted. Consider the Lyapunov function candidate

$$V = \frac{1}{2}\sum_{i=1}^{n}\left[z_i^2 + \left(\tilde{\theta}_i^s\right)^\top \Gamma_i^{-1}\tilde{\theta}_i^s\right] + \frac{1}{2\gamma_\psi}\tilde{\psi}^2.$$

By a back-stepping design procedure similar to (39)–(41), and after some algebraic manipulation, we have

$$\begin{aligned} \dot{V} &= \sum_{k=1}^{n}\left(\left(\tilde{\theta}_k^s\right)^\top \Gamma_k^{-1} - \sum_{l=k}^{n-1} z_{l+1}\frac{\partial \alpha_l}{\partial \hat{\theta}_k^s}\right)\left(\dot{\hat{\theta}}_k^s - \tau_{kn}\right) \\ &\quad - \sum_{k=1}^{n}\left[c_k z_k^2 + \sigma\left(\tilde{\theta}_k^s\right)^T\left(\hat{\theta}_k^s - \theta_k^0\right)\right] + \Lambda_n \\ &\quad + z_n\left[\phi_0 u - \alpha_n - y_r^{(n)}\right] \end{aligned}$$

where Λ_n is defined recursively as

$$\Lambda_1 \triangleq z_1\left[\rho_1 + (\beta_1 - 1)(\theta_1^s)^\top g_1^s(x_1) + \eta_1\right] + \gamma_\psi^{-1}\tilde{\psi}\dot{\hat{\psi}} \tag{59}$$

$$\begin{aligned} \Lambda_i &\triangleq \Lambda_{i-1} + z_i\left\{-\sum_{k=1}^{i-1}\frac{\partial \alpha_{i-1}}{\partial x_k}\left[(\beta_k - 1)(\theta_k^s)^\top g_k^s(\bar{x}_k) + \eta_k\right]\right. \\ &\quad \left. + \rho_i + (\beta_i - 1)(\theta_i^s)^\top g_i^s(\bar{x}_i) + \eta_i - \frac{\partial \alpha_{i-1}}{\partial \hat{\psi}}\dot{\hat{\psi}}\right\} \\ &\qquad \text{for } i = 2, \ldots, n. \end{aligned} \tag{60}$$

We choose the controller $u_I^s(t)$ defined in (4) and adaptive laws for updating $\hat{\theta}_k^s(t)$ as follows:

$$u_I^s(t) = \frac{1}{\phi_0}\left[\alpha_n + y_r^{(n)}\right] \tag{61}$$

$$\dot{\hat{\theta}}_k^s(t) = \tau_{kn}, \qquad 1 \leq k \leq n. \tag{62}$$

This operation yields

$$\dot{V} = -\sum_{k=1}^{n}\left[c_k z_k^2 + \sigma\left(\tilde{\theta}_k^s\right)^\top\left(\hat{\theta}_k^s - \theta_k^0\right)\right] + \Lambda_n \tag{63}$$

We now reconsider the adaptive bounding design procedure. First, by using (59), we have

$$\Lambda_1 \leq z_1\rho_1 + \gamma_\psi^{-1}\tilde{\psi}\dot{\hat{\psi}} + |z_1|\psi_m s_1(x_1) + |z_1|\bar{\eta}_1$$

where $s_1(x_1) \triangleq |g_1^s(x_1)|$. By choosing the bounding control function ρ_1 and the intermediate adaptation function ν_1 as $\rho_1 \triangleq -\hat{\psi}w_1 - \zeta_1\tanh(z_1\zeta_1/\epsilon), \nu_1 \triangleq \gamma_\psi[z_1 w_1 - \sigma(\hat{\psi} - \psi^0)]$, with $w_1 \triangleq s_1\tanh(z_1 s_1/\epsilon), \zeta_1 \triangleq \bar{\eta}_1$, and by using the property of the hyperbolic tangent function described by (48), we obtain

$$\Lambda_1 \leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + k\epsilon\psi_m + k\epsilon + \gamma_\psi^{-1}\tilde{\psi}(\dot{\hat{\psi}} - \nu_1).$$

For $2 \leq i \leq n$, from (60) and via mathematical induction, it follows that

$$\begin{aligned} \Lambda_i &\leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + (i-1)k\epsilon\psi_m + (i-1)k\epsilon + z_i\rho_i \\ &\quad + \left(\gamma_\psi^{-1}\tilde{\psi} - \sum_{k=1}^{i-2} z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\psi}}\right)(\dot{\hat{\psi}} - \nu_{i-1}) + \psi_m|z_i|s_i \\ &\quad - z_i\frac{\partial \alpha_{i-1}}{\partial \hat{\psi}}\dot{\hat{\psi}} + |z_i|\left(\bar{\eta}_i + \sum_{k=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_k}\right|\bar{\eta}_k\right) \end{aligned} \tag{64}$$

where $s_i \triangleq |g_i^s(\bar{x}_i)| + \sum_{k=1}^{i-1}|(\partial\alpha_{i-1})/(\partial x_k)||g_k^s(\bar{x}_k)|$.

The bounding control function ρ_i and the intermediate adaptation function ν_i are chosen to be

$$\begin{aligned} \rho_i &\triangleq -\hat{\psi}w_i + \frac{\partial \alpha_{i-1}}{\partial \hat{\psi}}\nu_i + \gamma_\psi w_i\sum_{k=1}^{i-2} z_{k+1}\frac{\partial \alpha_k}{\partial \hat{\psi}} \\ &\quad - \zeta_i\tanh(z_i\zeta_i/\epsilon) \end{aligned} \tag{65}$$

$$\nu_i \triangleq \nu_{i-1} + \gamma_\psi z_i w_i \quad w_i \triangleq s_i\tanh(z_i s_i/\epsilon) \tag{66}$$

$$\zeta_i \triangleq \bar{\eta}_i + \sum_{k=1}^{i-1}\left|\frac{\partial \alpha_{i-1}}{\partial x_k}\right|\bar{\eta}_k. \tag{67}$$

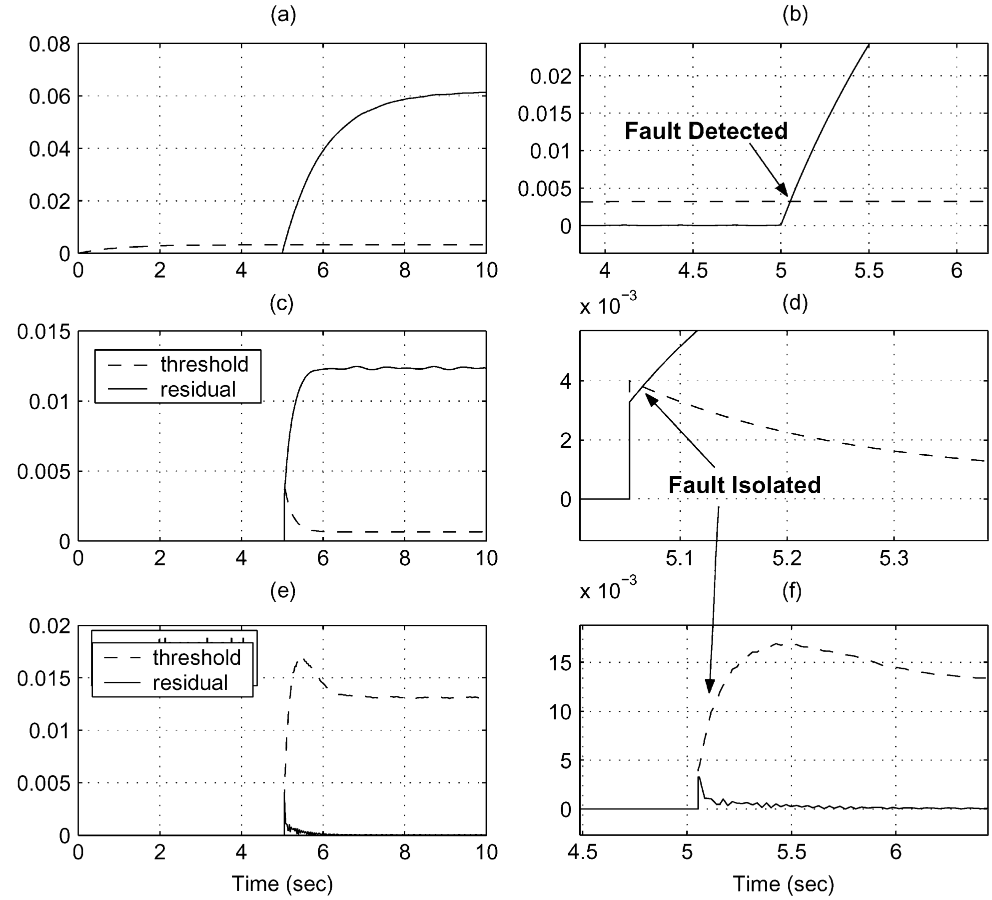

Fig. 3. (a) Behaviors of the fault detection residual (solid line) and of the threshold (dashed line). (b) Fig. (a) in enlarged form. (c) Behaviors of the fault isolation residual of estimator 1 (solid line) and of the threshold (dashed line). (d) Fig. (c) in enlarged form. (e) Behaviors of the fault isolation residual of estimator 2 (solid line) and of the threshold (dashed line). (f) Fig. (e) in enlarged form.

By using (64)–(67) recursively, for $i = n$, we finally have

$$\Lambda_n \leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + nk\epsilon\psi_m + nk\epsilon + (\dot{\hat{\psi}} - \nu_n) \cdot \left(\gamma_\psi^{-1}\tilde{\psi} - \sum_{k=1}^{n-1} z_{k+1}\frac{\partial\alpha_k}{\partial\hat{\psi}}\right).$$

Thus, by choosing the adaptive law

$$\dot{\hat{\psi}} = \nu_n \tag{68}$$

we obtain

$$\Lambda_n \leq -\sigma\tilde{\psi}(\hat{\psi} - \psi^0) + nk\epsilon\psi_m + nk\epsilon. \tag{69}$$

By substituting (69) into (63), it follows that $\dot{V} \leq -\bar{c}V + \bar{b}$, where $\bar{c} \triangleq \min_{1\leq i\leq n} \min[2c_i, \sigma/(\lambda_{\min}(\Gamma_i^{-1})), \sigma\gamma_\psi]$ and $\bar{b} \triangleq nk\epsilon\psi_m + nk\epsilon + (\sigma/2)(\sum_{k=1}^{n} |\theta_k^s - \theta_k^0|^2 + |\psi_m - \psi^0|^2)$.

By using the same arguments as in Section IV-B, we can state the following theorem.

Theorem 3: Assume that fault s occurs at time T_0 and that it is isolated at time T_{isol}^s. Then, the adaptive fault-tolerant control law (61), the fault parameter adaptive law (62), and the bounding parameter adaptive law (68) guarantee that:

1) all the signal and parameter estimates are uniformly bounded, i.e., $z(t), \hat{\theta}^s(t), \hat{\psi}(t)$, and $x(t)$ are bounded for all $t > T_{\text{isol}}^s$;

2) given any $\bar{\epsilon} > (2\bar{b}/\bar{c})^{1/2}$, there exists some $T(\bar{\epsilon})$ such that $|y(t) - y_r(t)| \leq \bar{\epsilon}$, for all $t \geq T(\bar{\epsilon})$. □

As we can see from the aforementioned design, the second fault-tolerant controller eliminates the limitations involved by Assumption 3 about the network approximation error bound in the design of the first fault-tolerant controller. Moreover, it is worth noting that the adaptive laws (46) and (62) for network weights and the fault parameters for fault accommodation are different from the adaptive laws (7) and (10) used for fault detection and isolation in Section III. This is not surprising as the objectives in these two cases are different: the goal of adaptive parameter estimation in the case of FDI is learning (i.e., approximating the fault function), whereas the objective of fault accommodation is to modify the feedback control law via parameter adaptation so as to stabilize the system and to guarantee some tracking performance in the presence of a fault. The presence of these two objectives at the same time is consistent with the structure depicted in Fig. 1, where it is pointed out that the monitoring module is important because: 1) it furnishes information on the current "health" of the controlled system; and 2) the information it provides through a learning algorithm can be exploited by the reconfigurable controller to achieve better tracking performances in the presence of a fault.

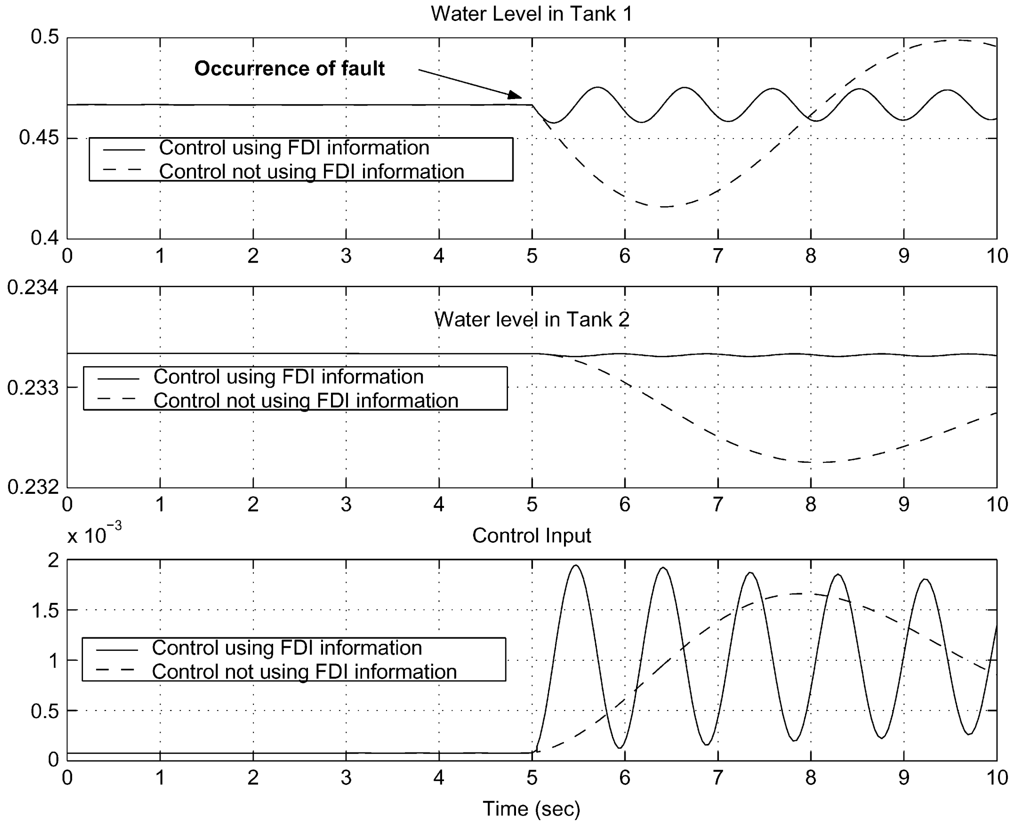

Fig. 4. Behaviors of the water levels in the two tanks and of the control variable under the action of the fault-tolerant controller (solid line) and under the action of the controller for which the second reconfiguration is not enabled (dashed line).

V. Simulation Results

We now give a simulation example to illustrate the effectiveness of the proposed fault-tolerant control methodology. In particular, we consider a well-known benchmark problem for fault-tolerant control. It deals with a laboratory process using two tanks with fluid flow [1]. The two tanks are identical and cylindrical in shape, with a cross section $A_s = 0.0154$ m^2. The cross section of the connection pipes is $S_p = 3.6 \cdot 10^{-5}$ m^2, and the liquid levels in the two tanks are denoted by h_1 and h_2, respectively. The supplying flow rates coming from a pump to tank 1 are denoted by u. There is an outflow from tank 2. By using balance equations and Torricelli's rule, we obtain the following equations:

$$\begin{aligned} \dot{h}_1 &= (-a_1 S_p \operatorname{sign}(h_1 - h_2)\sqrt{2g|h_1 - h_2|} + u)/A_s \\ \dot{h}_2 &= (a_1 S_p \operatorname{sign}(h_1 - h_2)\sqrt{2g|h_1 - h_2|} \\ &\quad -a_2 S_p \sqrt{2gh_2})/A_s \end{aligned} \tag{70}$$

where $a_1 = 1$ and $a_2 = 1$ denote nondimensional outflow coefficients, and g is the gravity acceleration. The fault class under consideration is defined as follows.

1) Leakage in tank T_1. We assume that the leak is circular in shape and of unknown radius r_1. Then, denoting by q_{1f} the outflow rate of the unknown-size leak in tank T_1, we have $q_{1f} = c_1 \pi (r_1)^2 \sqrt{2gh_1}$.

2) Leakage in tank T_2. By analogy to the case of the leakage in tank T_1, we have $q_{2f} = c_2 \pi (r_2)^2 \sqrt{2gh_2}$.

By linearizing the nominal system model described by (70) at a trim condition of $h_1^e = 0.4667$ m, $h_2^e = 0.2333$ m, and $u^e = 7.7027 \cdot 10^{-5}$ m^3/s, and including the effects of faults and modeling uncertainty, we have

$$\begin{aligned} \dot{x}_1 &= -0.0214 x_1 + 0.0107 x_2 + \beta_1 f_1 \\ \dot{x}_2 &= 0.0107 x_1 - 0.0107 x_2 + 64.9351 \bar{u} + \eta_2 + \beta_2 f_2 \end{aligned}$$

where $x_1 = h_2 - h_2^e, x_2 = h_1 - h_1^e$, and $\bar{u} = u - u^e, \eta_2$ represent modeling uncertainty, β_1 and β_2 are fault time profile functions, and f_1 and f_2 are fault functions given by[2]

- $$f_1 = \theta^1 g^1(x), \theta^1 \triangleq (r_2)^2,\ g^1(x) \triangleq (1/A_s) c_2 \pi \sqrt{2g\,(x_1 + h_2^e)}$$
- $$f_2 = \theta^2 g^2(x), \theta^2 \triangleq (r_1)^2,\ g^2(x) \triangleq (1/A_s) c_1 \pi \sqrt{2g\,(x_2 + h_1^e)}.$$

We consider the case of abrupt faults (the case of incipient faults is completely analogous and is not addressed here for the sake of brevity). More specifically, in tank 1 a leakage of $r_1 = 0.01$ m is assumed to occur at $t = 5$ s. The modeling uncertainty used is $(\sin(10t))/(A_s) \cdot 10^{-5}$, which gives a bound of $5 \cdot 10^{-5}/A_s$. Using the methodology described in Section III, the monitoring module consisting of an FDAE and a bank of two FIE's are designed. The design parameters are taken to be $\lambda^0 = 1, \lambda^1 = \lambda^2 = 5$.

[2]Note that, after the linearization procedure, in order to write the state equations consistently with the general form (13), the state variable x_1 was associated with h_2 and the state variable x_2 was associated with h_1. As a consequence, the fault function f_1 turns out to be associated with the leakage in tank 2 and the fault function f_2 with the leakage in tank 1.

The control objective is to maintain the fluid level in tank 2 at a reference value. The controller module is determined by the algorithms described in Section IV. Specifically, the online approximation model in the first fault-tolerant controller design is implemented as a radial-basis-function neural network with five fixed centers evenly distributed in the interval $[-0.05, 0.05]$. The width of each RBF is 0.04. For simplicity, the bound on the network approximation error is taken to be an unknown constant ψ_δ. The design parameters are selected as follows: the controller gains $c_1 = c_2 = 0.02, \sigma = 0.01, \epsilon = 0.1, \theta^0 = 0, \psi^0 = 0, \gamma_\psi = 0.01$, and $\Lambda = 0.01$. As shown in Fig. 3, the fault is immediately detected and isolated after it occurs at $t = 5$ s.

Regarding the performances of the fault-tolerant controller, in Fig. 4 we compare the behavior of the two-tank system under the action of the proposed FTC with the behavior of the system when the second-controller reconfiguration is not enabled (e.g., the controller does not take advantage of the isolation information). As can be seen, the recovery of the tracking performances is significantly worse when the isolation information is not exploited to activate the second-controller reconfiguration. Indeed, the proposed fault-tolerant controller makes it possible to approximately maintain the level of tank 2 at a reference value of 0.2333 m, even after the occurrence of the fault.

VI. Conclusion

In recent years, there has been significant research activity in the fault diagnosis (fault detection and isolation) area and the fault-tolerant control area. However, links between these two areas are still lacking. In this paper, we have presented a unified methodology for fault diagnosis and accommodation in a class of nonlinear systems. The proposed FTC architecture consists of an on-line monitoring module used to detect and to isolate faults, and a controller module to accommodate the effects of faults on the basis of the fault information obtained by the fault-diagnosis procedure. An adaptive tracking design has been developed that uses neural networks to approximate the unknown fault function. The fault-tolerance has been enhanced by use of adaptive bounding design techniques. Under certain assumptions, the stability of the proposed robust FTC scheme has been rigorously established by using the Lyapunov synthesis approach.

The extension of the proposed integrated approach to fault diagnosis and controller reconfiguration to a larger class of nonlinear systems and faults deserves further research. Moreover, considerable effort is devoted to generalizing the methodology to the case where not all state variables are available for measurements; this is clearly very important from an application point of view.

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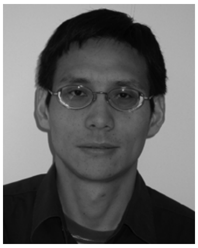

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